Senior 3 Applied Mathematics Exercises

A Supplement to
A Foundation for
Implementation

Renewing Education:
New Directions

Manitoba
Education
and Training
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CONTENTS

Unit A: Non-Linear Functions *A-1*
Non-Linear Functions — Solutions *A-13*

Unit B: Personal Finance *B-1*
Personal Finance — Solutions *B-21*

Unit C: Systems of Linear Equations *C-1*
Systems of Linear Equations — Solutions *C-11*

Unit D: Linear Programming *D-1*
Linear Programming — Solutions *D-13*

Unit E: Budgets and Investments *E-1*
Budgets and Investments — Solutions *E-15*

Unit F: Data Management and Analysis *F-1*
Data Management and Analysis — Solutions *F-41*

Unit G: Precision Measurement *G-1*
Precision Measurement — Solutions *G-17*

Unit H: Geometry *H-1*
Geometry — Solutions *H-19*

Note: Unit problems can be copied and handed to students as assignments. Teaching information is provided with the solutions.

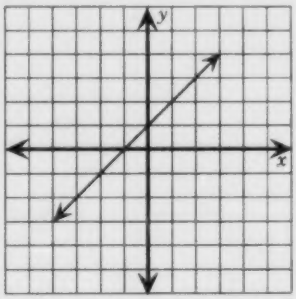
Unit A
Non-Linear Functions

Teachers are advised to refer to *Senior 3 Applied Mathematics: A Foundation for Implementation* as their ***primary*** source of suggestions for instruction and assessment. The following exercises are an excellent resource if supplementary material is required.

Exercise 1: Quadratic Functions

1. Identify the relations as linear functions, quadratic functions, or other.

a)



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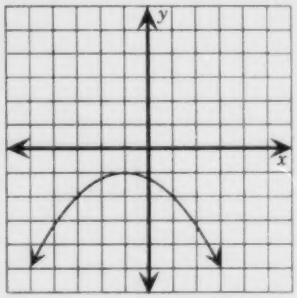

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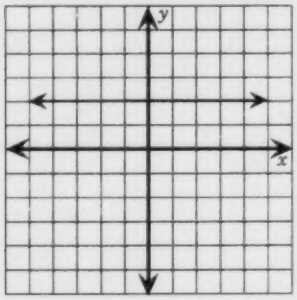

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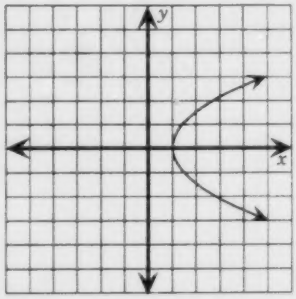

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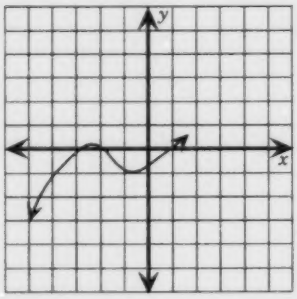

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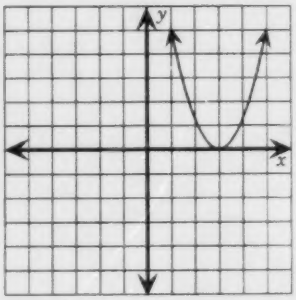

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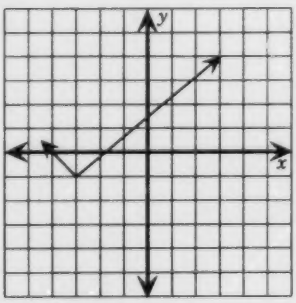

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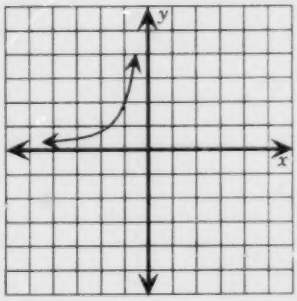
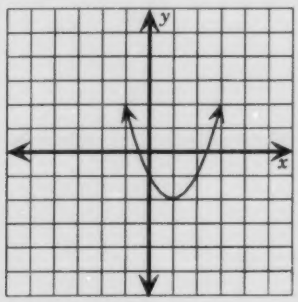


Exercise 1: Quadratic Functions (continued)

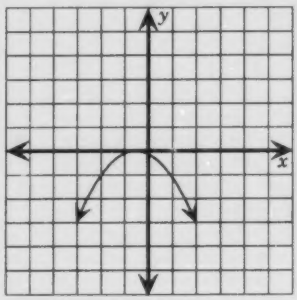
2. Identify the relations as linear functions, quadratic functions, or other.
   a) $y = x^2 + x$
   b) $y = 5x + 3$
   c) $x + y = x^3 + x^2$
   d) $x + y = x^2 + 1$
   e) $x^2 + y^2 = 9$

3. Identify (i) the coordinates of the vertex, (ii) the x-intercepts, (iii) the domain, and (iv) the range of each quadratic relation. Round all answers to one decimal place.

   a)



   b)



   c) $y = x^2 + 6x + 4$
   d) $y = 4 - x^2$

4. Use a graphing utility (computer or calculator) to determine the coordinates of the vertex. Round all answers to one decimal place.

   a) $y = x^2$
   b) $y = -x^2 + 5x + 4$
   c) $y = -x^2 + 4x$ Note: $y = -1(x^2) + 4x$.
   d) $x + y = x^2 + 1$
   e) $y = (x - 2)^2$
   f) $y = (x + 2)(x - 5)$
   g) $y = x(x + 6)$
   h) $y = -x^2 - 2$
   i) $y = \frac{x^2}{2} + \frac{x}{3} - \frac{1}{4}$

5. Sketch the graph of a quadratic function that has
   a) a maximum value of $y = 8$ and the x-intercepts $x = 2$ and $x = 6$
   b) a minimum value of $y = -4$ and the x-intercepts $x = -3$ and $x = 1$
   c) What are the coordinates of the vertex in (a)? In (b)?

Exercise 1: Quadratic Functions (continued)

6. Observe the graphs of the quadratic relations shown. How can you predict whether the graphs will have a maximum or a minimum value (that is, how can you predict that the graphs will be "cap" shaped or "cup" shaped)?

   a) $y = x^2$

   b) $y = -x^2$

   c) $y = 2x^2 + 1$

   d) $y = -2x^2 + 1$

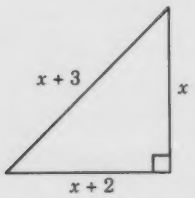
7. Determine whether

   a) the point (5, 70) is or is not a point on the curve described by $y = 2x^2 + 3x + 4$.

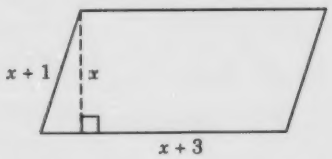
   b) the curve of the function $y = x^2 - 4$ intersects the x-axis.

8. Write an appropriate expression for the area of the following figures.

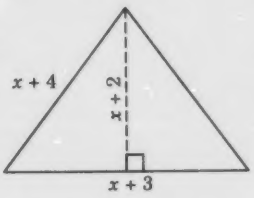
   a)



   b) parallelogram

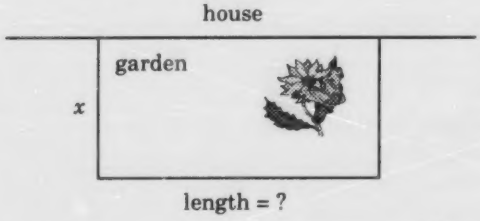


   c) isosceles triangle



Exercise 1: Quadratic Functions (continued)

9. Jane has 24 metres of chain link fence to put around her garden. She needs to keep the neighbours out of her garden! The garden is next to her house, and so the fence encloses only three sides of the garden. She wants the garden to be as large as possible. The purpose of this question is to find the dimensions of the garden that will have the largest area.



a) Produce a table with columns for width, length, perimeter, and area (as shown). Use a spreadsheet if possible.

i) What is the variable representing the width?

ii) What is a suitable expression representing the length of the garden? (x)

iii) What is the equation representing the area of the garden? (y)

	A	B	C	D
1	width (m)	length (m)	perimeter (m)	area (m^2)
2	0	x	24	y
3	1		24	
4	2		24	
5	3		24	
6	4		24	
7				

Possible spreadsheet formulas:
x =C2 – 2*A2
y = A2*B2

b) Draw a graph of width versus area. Draw the graph in such a way that the area (y) depends on the width (x). If possible, use the graphing function of the spreadsheet, or a graphing calculator.

i) What is the shape of the graph? Name the type of function this graph describes.

ii) What are the coordinates of the vertex of the graph? Include units with your answer.

iii) State the domain and range of the graph. (Is it possible for the width or area to be less than zero?)

iv) What is the equation of the axis of symmetry?

v) What are the x-intercepts of the graph? What is the significance of the x-intercepts?

vi) What is/are the y-intercept(s) of the graph? What is the significance of the y-intercept(s)?

vii) What is the maximum area that can be enclosed with the 24-metre fence?

c) What would be the maximum area possible if a 48-metre fence were used instead of the 24-metre fence? Would the area be twice as large? What would be the area if a 40-metre fence were built? Explain how you arrive at your answers.

Exercise 1: Quadratic Functions (continued)

10. A ball is thrown vertically (with a pitching machine) at an initial speed of 100 miles per hour (that is about 160 km/hr). The height, h, of the ball at any time, t, is given by the function $h = 147t - 16t^2$, where the height is measured in feet and the time in seconds.

    a) Find the maximum height that the ball will fly.

    b) When does the ball reach the maximum height?

    c) When does the ball hit the ground?

    d) What is the height of the ball one second after being thrown?

11. A football that is kicked for a field goal flies in a path described by the equation $y = \frac{4x}{3} - \frac{x^2}{90}$.

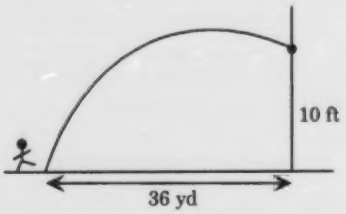
    The ball is kicked from the 35 yard line. In this equation, the height of the ball is y, and x is the horizontal distance (in feet) from the kicker. Round all answers to the nearest foot.

    a) What is the maximum height of the football?

    b) How far (horizontally) will the ball fly before it hits the ground?

    c) Will the ball clear the crossbar? The crossbar is 10 feet above the playing surface.

    d) How far will the ball be above/below the crossbar?



12. A helicopter shuttle service operating between an airport and the centre of a city charges a fare of $10.00 and carries 300 persons per day. The manager estimates that he will lose 15 passengers for each increase of $1.00 in fare. Find the most profitable fare to charge.

13. At a certain service station, an average of 4000 litres of gasoline is sold daily at 50¢ per litre. The owner estimates that he will sell 60 litres fewer per day for every 1¢ increase in the price per litre. Find the price (rounded to the nearest cent) that will give the owner maximum daily receipts. What are the maximum receipts?

Exercise 2: Cubic Functions

1. Use a graphing utility to draw the graphs of these cubic functions. Write (i) the coordinates of the relative minimum or maximum values, if any. Also, (ii) identify any x- and y-intercepts. Round your answers to 1 decimal place.

   a) $y = x^3 - 9x^2 + 23x - 15$

   b) $y = 2x^3 - 5x^2 + x - 3$

   c) $y = -3x^3 + 4x^2 - 2x + 3$

   d) $f(x) = x^3 + 3x^2 + 3x + 1$

   e) $y = (x-1)^2(x+2)$

   f) $f(x) = (x-1)^2(x+3)$

   g) $y = (x+3)(x-2)(x+1)$

   h) $f(x) = \frac{1}{3}x^3 - \frac{5}{2}x^2 + 6x - 1$

2. A variety of open top boxes are made from a 20-cm by 30-cm rectangular flat sheet by cutting equally sized squares off of each corner of the sheet and folding up the four sides. How long should the squares be to produce a box of maximum volume? What is the maximum volume?

3. A box with an open top is to be built from a square piece of cardboard, 3 feet wide, by cutting out a square from each of the four corners and bending up the sides. Find the largest volume that the box can have.

Exercise 3: Exponential Functions

1. Use a graphing utility to determine the following graphs.

a) Sketch, on the same coordinate plane, the following graphs of $y = b^x$ when $b > 1$.

i) $y = 2^x$

ii) $y = 3^x$

iii) $y = 5^x$

iv) $y = 10^x$

v) $y = (1.5)^x$

b) Sketch, on the same coordinate plane, the following graphs of $y = b^x$ when $0 < b < 1$.

i) $y = (0.8)^x$

ii) $y = (0.5)^x$

iii) $y = (0.2)^x$

iv) $y = (0.05)^x$

c) Sketch, on the same coordinate plane, the following graphs of $y = b^x$ when $b < 0$.

i) $y = (-2)^x$

ii) $y = (-3)^x$

iii) $y = (-10)^x$

d) Sketch, on the same coordinate plane, the following graphs of $y = a(b)^x$ when $b = 2$ and $a > 1$.

i) $y = 2(2)^x$

ii) $y = 3(2)^x$

iii) $y = 4(2)^x$

e) Sketch, on the same coordinate plane, the following graphs of $y = a(b)^x$ when $b = 2$ and $0 < a < 1$.

i) $y = 0.2(2)^x$

ii) $y = 0.5(2)^x$

iii) $y = 0.9(2)^x$

f) Sketch, on the same coordinate plane, the following graphs of $y = a(b)^x$ when $b = 2$ and $a = 0$.

i) $y = -1(2)^x$

ii) $y = -2(2)^x$

iii) $y = -0.5(2)^x$

Exercise 3: Exponential Functions (continued)

2. **M & M Growth**

Materials 40 M&Ms per student/group, paper cup

Problem What is the relationship between the number of M&Ms in the cup and the number of trials?

Directions

Each student/group puts 4 M&Ms into the cup. This is trial no. 0.

Pour the M&Ms onto the table. You count the number of candies that have the M showing. For every M showing, you add one more candy to the cup. Then return all the spilled M&Ms to the cup with the extra ones added. The new total is trial no. 1.

You now pour the M&Ms onto the table and repeat the counting and adding process to produce data for trial no. 2.

This process is continued until you run out of candies.

Record the data of the other groups in the class, or repeat the experiment several times.

Data

	Group 1	Group 2	Group 3
Trial No.	No. of M&Ms	No. of M&Ms	No. of M&Ms
0	4	4	4
1			
2			
3			
4			
5			
6			
7			

a) Draw a sketch showing the relation between the number of candies (on the y-axis) and the trial number (on the x-axis). The graph should be drawn with a graphing utility. The graph shows an exponential growth curve that can be represented by the relation $y = a(b)^x$.

b) Use an exponential regression to determine the values of a and b in the relation $y = a(b)^x$.

c) What is the significance (in question b) of the value of b? Of the value of a?

d) How could you change the experiment so that the relation would be $y = 200(b)^x$?

e) How would you change the experiment so that the relation would be $y = a(0.25)^x$ or $y = a(1/4)^x$?

f) How would the equation and the shape of the graph change if half of the candies had an M on one side, and the other half of the candies had an M on both sides?

Exercise 3: Exponential Functions (continued)

3. **Final Value of an RRSP**

A bank account pays interest at the rate of 5% per annum compounded annually for a long-term RRSP. Use a spreadsheet (such as has been started below) to demonstrate the growth in value of the RRSP over 20 years if the initial value is $200.

	A	B	C
1	Time (yr)	Value ($)@5%	
2	0	200	
3	1	**	
4	2	***	
5	3		
6	*		

* = A5 + 1

Note: After 1 year the value of the RRSP will be the initial value plus interest

$y = 200.00 + 0.05(200.00) = 200.00(1 + 0.05) = 200(1.05)$

$y = \$210.00$

** y = B2*1.05

At the end of the second year the value of the RRSP will be $210.00 plus interest.

$y = 210.00 + 0.05(210.00) = 210.00(1 + 0.05) = 210(1.05)$

$y = 200(1.05)(1.05)$

$y = 200(1.05)^2$

$y = 220.50$

*** y = B2*1.05^A4

Use "Fill Down" feature to complete columns A and B.

Note the similarity:
$y = 200(1.05)^2$
$y = a(b)^x$

— continued

Exercise 3: Exponential Functions (continued)

Final Value of an RRSP (continued)

Use the graphing feature of the spreadsheet to draw the graph of the value (y) versus the time (x). If this is not available, then use a graphing utility to sketch the graph of $y = 200(1.05)^x$ and answer the questions.

a) What are the values of a and b? What is the significance of each? What do x and y represent? (You may want to compare these answers to a previously used formula, $A = P(1 + i)^n$.)

b) Use a scientific calculator to determine the value of the RRSP after 10 years. After 20 years.

c) Approximately how many years are required for the RRSP to double in value? This answer may be observed from the graph.

d) Add another column to the spreadsheet. Show the value of the RRSP if the rate of return is 6% per annum compounded annually. Draw the graphs of the values at 5% and 6% versus the time (20 years) onto the same coordinate plane. What is the difference in the values of the RRSP after 20 years if invested at 6% rather than 5%?

e) Use the exponential formula $y = a(b)^x$ to solve the following. You will first need to determine the correct values of a, b, and x to solve for y. Use the information from your answer to question (a). Find the final value of each:

i) $800.00 invested for 15 years at 9% per annum compounded annually

ii) $2000.00 invested for 10 years at 12% per year compounded annually

iii) $2000.00 invested for 10 years at 12% per year compounded semiannually

iv) $2000.00 invested for 10 years at 12% per year compounded quarterly

v) $2000.00 invested for 10 years at 12% per year compounded monthly.

Unit A
Non-Linear Functions
Solutions

Exercise 1: Quadratic Functions — Solutions

1. a) linear
   b) quadratic
   c) linear
   d) other
   e) other
   f) quadratic
   g) other
   h) other

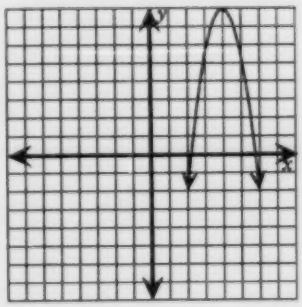
2. a) quadratic
   b) linear
   c) other
   d) quadratic
   e) other

3.

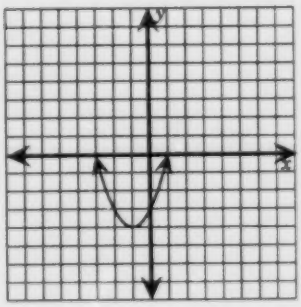
	vertex	x-intercept	domain	range
a	(1, –2)	{–0.6, 2.5}	Real No.	$y \geq -2$
b	(–0.5, 0)	{–0.5}	Real No.	$y \leq 0$
c	(–3, –5)	{–0.8, –5.2}	Real No.	$y \geq -5$
d	(0, 4)	{2, –2}	Real No.	$y \leq 4$

4. a) (0, 0)
   b) (–2.5, –2.3)
   c) (4, 2)
   d) (0.5, 0.8)
   e) (2, 0)
   f) (1.5, –12.2)
   g) (–3, –9)
   h) (0, –2)
   i) (–0.3, –0.3)

5. a)



b)



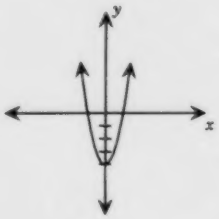
c) Graph (a): (4, 8); Graph (b): (–1, –4)

Exercise 1: Quadratic Functions — Solutions (continued)

6. When the number in front of x^2 is positive, the function has a minimum value.
   When the number in front of x^2 is negative, the function has a maximum value.

7. a) Substitute (5, 70) into the equation: $70 = 2(5)^2 + 3(5) + 4$. Since $70 \neq 69$, (5, 70) is not a point on the curve.

   b) Sketch $y = x^2 - 4$



   $\therefore$ the function intersects the x-axis

8.
   a) $A = \dfrac{x(x+2)}{2}$

   b) $A = x(x+3)$

   c) $A = \dfrac{(x+3)(x+2)}{2}$

9. a) i) x
      ii) $L = 24 - 2x$
      iii) $A = x(24 - 2x)$

   b) i) parabola, quadratic function
      ii) V(6 m, 72 m^2)
      iii) domain: $(0 \leq l \leq 12)$ m; range: $(0 < A < 72)$ m^2
      iv) $x = 6$
      v) $x = L = 0$ m, 12 m
      vi) $y = A = 72$ m^2
      vii) $A = 72$ m^2

   c) $A = 288$ m^2

10. a) $h = 337.6$ ft
    b) $t = 4.6$ sec
    c) $t = 9.2$ sec
    d) $h = 131$ ft

11. a) $h = 40$ ft
    b) $d = 120$ ft = 40 yd
    c) yes
    d) 4.4 ft above

12. Let x = fare increase (in dollars); $y = (300 - 15x)(10 + x)$; $x = \$15.00$

13. Let x = price increase (in cents); $y = (4000 - 100x)(50 + x)$; price = 58¢; max. profit = \$2041

Exercise 2: Cubic Functions — Solutions

1. a) max (1.8, 3.1); min (4.2, –3.1); x = {1.0, 3.0, 5.0}; y = {–15.0}
   b) max (0.1, –2.9); min (1.6, –2.9); x = {2.5}; y = {–3.0}
   c) no max or min; x = {1.4}; y = {3.0}
   d) no max or min; x = {–1.0}; y = {1.0}
   e) max (–1.0, 4.0); min (1.0, 0.0); x = {–1.0, 2.0}; y = {2.0}
   f) max (–1.6, 9.5); min (1.0, 0.0); x = {1.0, –3.0}; y = {3.0}
   g) max (–2.1, 4.1); min (0.8, –8.2); x = {–3.0, –1.0, 2.0}; y = {–6.0}
   h) max (–2.0, 8.3); min (2.0, –2.3); x = {–3.8, 0.8, 3.0}; y = {3.0}
   i) max (2.0, 3.7); min (3.0, 3.5); x = {0.2}; y = {–1.0}
2. x = 3.9 cm; V = 1056 cm^3
3. V = 2 ft^3

Exercise 3: Exponential Functions — Solutions

Answers will vary.

Unit B
Personal Finance

Teachers are advised to refer to *Senior 3 Applied Mathematics: A Foundation for Implementation* as their ***primary*** source of suggestions for instruction and assessment. The following exercises are an excellent resource if supplementary material is required.

Exercise 1: Income

1. Compute the weekly gross earnings for these employees. Time and a half is paid for all hours over 40 they work in a given week.

Employee Name	Hourly Rate	Mon.	Tues.	Wed.	Thurs.	Fri.
a) J. MacAdoo	$10.20	8	9	10	11	7.5
b) S. Kashin	$14.15	7	7	12	10	8
c) P. Dyck	$20.00	8	8	9	12	9

2. Compute the weekly gross earnings for these employees. Time and a half is paid for all hours over 8 they work in a given day.

Employee Name	Hourly Rate	Mon.	Tues.	Wed.	Thurs.	Fri.
a) J. Rees	$8.20	9	8.5	7	10	7.5
b) S. L'Heureux	$12.25	8	9.5	10	9	8.5
c) P. Bennett	$18.00	10	8	9	11	9.5

After determining gross pay, the payroll department calculates net pay after deductions. All working Canadians over 18 must pay into the Canada Pension Plan (CPP), the Employment Insurance plan (EI), and income tax. Additionally, most Canadians will have other deductions from their pay. Some of these include union dues, company pension plan, group life insurance, longterm disability, RRSP contributions, and charitable donations.

In this section you will calculate gross pay and then determine net pay assuming the employee has to pay CPP, EI, and Income Tax.

To determine the above three deductions, you can use

1. Payroll Deductions Tables (paper copy) produced annually by Revenue Canada
2. Payroll Deductions Tables on diskette produced annually by Revenue Canada
3. Formulas that calculate these deductions. (These formulas are found in the Payroll Deductions Tables booklet from Revenue Canada.)

The illustrated question below uses the 1998 Payroll Deductions Tables from Revenue Canada. Use the tables for the current year when calculating net pay.

Exercise 1: Income (continued)

Example

A person worked 50 hours at $15.00 per hour. Overtime is paid after 40 hours in a week. Determine the person's net income if the claim code is 5 and he or she must pay CPP and EI and these are the only deductions from payroll.

Step 1: Calculate Gross Pay:

Regular Pay	$15.00 x 40 = $600.00
Overtime Pay	$15.00 x 10 x 1.5 = $225.00
Gross Pay	$600.00 + $225.00 = $825.00

Step 2: Use tables to find for this level of income CPP, EI, and income tax.

CPP:	$24.24
EI	$22.28

Previously, CPP and EI were deducted from gross pay to arrive at a figure called Taxable Earnings. This is no longer the case. Union dues and company pension plan deductions are allowed but not EI and CPP deductions. Therefore, you need to look up $825.00 under claim code 5 of the Weekly Pay Period Table to find the amount of tax.

Income Tax	$200.25

Step 3: Determine net pay:

Gross Pay – CPP – EI – Income Tax = Net Pay

$825.00 – $24.24 – $22.28 – $200.25 = $578.23

Use the above sequence to determine the worker's net pay for each question that follows:

3. One particular week an assembly line worker worked 54 hours (40 were regular hours, 6 hours were paid at time and a half, and the remainder at double time). Find the worker's net pay if the regular rate of pay was $14.00 per hour and the claim code is 4.

4. A salesperson receives 8% commission on the first thousand dollars of sales, and 15% on all sales in excess of $1000. If sales for the past week amounted to $5000, what was the salesperson's total commission? Determine the worker's net pay after deductions if the claim code is 3.

5. A salesclerk in an appliance section of a department store receives a regular salary of $250 a week plus 5% commission on sales in excess of $900. Last week's sales amounted to $3150. What was the salesclerk's total earnings for the week? Determine the worker's net pay after deductions if the claim code is 5.

6. A waiter earns $5.75 an hour for a 40-hour week. He makes time and a half for overtime. One week he worked 45 hours and made $185 in tips. Find his gross pay for the week. Determine the worker's net pay after deductions if his claim code is 4.

7. Fred's monthly salary is $700. In addition, he receives 5% commission rate on the first $12 000 of his sales and 7% commission rate on all sales over $12 000. Last month Fred sold $24 000 worth of products. What was his gross pay? Determine Fred's net pay after deductions if his Claim Code is 2.

Exercise 1: Income (continued)

8. Two friends were comparing the different pay scales paid by the two companies for which they worked.

   Company A: Paid employees overtime after 40 hours worked in a week
   Company B: Paid employees overtime after 8 hours worked in a day

   Suppose they worked the following hours every day during the week. Compare the total pay between company A and company B if the employees earned $16.00 per hour. Determine their net pay after deductions if both friends have a claim code of 4.

Monday	Tuesday	Wednesday	Thursday	Friday
11	7	11	12	11

9. Charlie has a job making wooden pallets. He earns $1.20 for each pallet made. Charlie averages 100 pallets in a typical working day. What will be his gross pay if he works 5 days in a week? Determine Charlie's net pay after deductions if his claim code is 3.

10. Prepare a spreadsheet that will automatically calculate the gross pay of an individual given the hours worked, the rate of pay, and the point where the worker begins earning overtime. For example, calculate the gross pay of an individual who worked 52 hours in a week at $16.00 per hour. Overtime is paid after 40 hours worked in a week and the overtime rate is 1.5 times the regular rate of pay.

Exercise 2: Property Taxation

1. The Gysel family owns a home assessed at $90 000. The rate of assessment is 45%. The mill rate is 62 mills with a local improvement tax of $180.00 for sidewalk reconstruction. What was the total tax bill for the family?

2. The Walchucks purchased a $120 000 home. At the time of purchase the home was assessed at $80 000 and the rate of assessment was 45%. A tax assessor reassessed the house at $90 000. The rate of assessment remained the same. Assuming a mill rate of 55 mills, find the amount of general tax increase resulting from the reassessment.

3. A ratepayer has his house assessed at $85 000. The rate of assessment is 45%. The lot has a 15-m frontage. Local improvements are charged at an annual rate per metre of frontage as follows: sewer $3.87/m and sidewalks $2.50/m. Find the ratepayer's tax bill before school taxes if the municipal mill rate is 70 mills.

4. The rate of property tax in mills for a municipality can be found using the following formula:

$$\text{Mill Rate} = \frac{\text{Total Tax to Be Raised}}{\text{Total Assessed Value of Property}} \times 1000$$

Calculate the rate of assessment in the rural municipality to the nearest whole mill for each of the years listed.

Year	Assessed Value of Real Property	Budget Requirement	Mill Rate
a) 1996	$780 000 000	$69 000 000	
b) 1997	$852 000 000	$82 000 000	
c) 1998	$945 000 000	$95 000 000	

Exercise 2: Property Taxation (continued)

5. Below is a partially completed "statement and demand for taxes." Given the information provided on the form, complete the amount of Total Taxes Due by the homeowner.

   Note: Be careful when calculating the levy on sewer and water. The amount is $70.00 plus 0.050 mills on the Total Port. Assessment. The other local improvements are calculated only on the Total Port. Assessment.

STATEMENT AND DEMAND FOR TAXES

REAL PROPERTY **ROLL NUMBER:**
PAGE: 1 OF 1

PROPERTY DESCRIPTION

ROLL NUMBER	WARD	Lot/Section	Blk/Twp	Plan/Range	Frontage/Area	Dwell. Units
4120		3	6	120	65.2 F	1

Civic Address:

Title or Deed No.	Current Assessment Land	Current Assessment Buildings	Status Code	Total Assessment	Prop. Class	Portion %	Total Port. Assessment
B630	9200	65 000	T	74 200	10	45	33 930

- ERRORS AND OMISSIONS EXCEPTED
- ALL LAND IN ARREARS FOR MORE THAN ONE YEAR SHALL BE SOLD FOR TAXES
- ALL CHEQUES MUST BE MADE PAYABLE IN CANADIAN FUNDS
- CASH REGISTER RECEIPT IMPRESSION OR BANK RECEIPT CONSTITUTES OFFICIAL RECEIPT
- RETAIN COPY FOR INCOME TAX PURPOSES

ASSESSMENT SUBJECT TO LOCAL IMPROVEMENT LEVY

MUNICIPAL TAXES

Description			Assessment	Mill Rate	Levy
General Municipal			?	22.5	?
By-Law No.	**Term**	**Type**	**Frontage Levy**	**Mill Rate**	**Levy**
412	98	Sewer & Water	70.00	0.050	?
506	99	Sidewalk		2.460	?
508	99	Street		3.150	?

EDUCATION TAXES

Description	Assessment	Mill Rate	Levy
Provincial Education 1	33 390	8.1	?
Provincial Education 2	33 390	21.5	?

PROVINCIAL TAX CREDITS

(See Manitoba Enclosure for Additional Information)	Description	Credit
	Manitoba Resident Homeowner Tax Assistance	250.00

TOTAL TAXES DUE

Municipal Taxes	Education Taxes	Total Current Taxes	Prov. Credits	Net Taxes	*Arrears/Credits	Added Taxes	Total Taxes Due
					ϕ	ϕ	

Exercise 2: Property Taxation (continued)

6. Create a spreadsheet template that would enable the user to do the following:
   a) calculate the total tax bill
   b) answer "what if" questions such as:
      — What happens to the tax bill if the mill rate increases or decreases?
      — What happens to the tax bill if the assessed value increases but the mill rate decreases (that is, changing assessed values)?
      — What happens to the tax bill if a local improvement is paid off and is no longer part of the tax bill?
      — If a municipality is considering paving streets, what impact on the tax bill would an increase of 5 mills have on the total tax bill?

   A homeowner has received the tax bill shown on the next page. Answer the following questions using your "what if" capabilities on the spreadsheet you created. Go back to your original template to answer each question.

   i) Assuming no changes except a rise in the mill rate to 25 mills from 21.580, what would be the new total taxes due?

   ii) Reassessment was done throughout Manitoba. The home now has an assessed value on the building of $69 000. Assuming no other changes in the tax form, what would be the impact on the total taxes due?

   iii) The local school board was facing a financial crisis. Finding no other solution, the board raised the Provincial Education 2 mill rate from 22.000 mills to 26.000 mills. Calculate the new total taxes due.

   iv) If the municipality were to pave more roads in the area and added a new by-law that created a new tax of 3.050 mills, what would be the increase in total taxes due? Some municipalities give their ratepayers the right to vote on this type of issue. What factors might determine whether this particular homeowner voted "Yes or No" when casting her ballot?

   v) At a recent municipal council meeting one councillor stated that "raising the mill rate one mill is no big deal because no one will notice." Calculate the change in total taxes due if the council raises the mill rate 1 mill.

Exercise 2: Property Taxation (continued)

STATEMENT AND DEMAND FOR TAXES

REAL PROPERTY **ROLL NUMBER:**
PAGE: 1 OF 1

PROPERTY DESCRIPTION						
ROLL NUMBER	WARD	Lot/Section	Blk/Twp	Plan/Range	Frontage/Area	Dwell. Units
39800		2	2	1447	78.18 F	1

Civic Address:

Title or Deed No.	Current Assessment Land	Current Assessment Buildings	Status Code	Total Assessment	Prop. Class	Portion %	Total Port. Assessment
A30954	8,900	61,200	T	70,100	10	45.00	31,550

- ERRORS AND OMISSIONS EXCEPTED
- ALL LAND IN ARREARS FOR MORE THAN ONE YEAR SHALL BE SOLD FOR TAXES
- ALL CHEQUES MUST BE MADE PAYABLE IN CANADIAN FUNDS
- CASH REGISTER RECEIPT IMPRESSION OR BANK RECEIPT CONSTITUTES OFFICIAL RECEIPT
- RETAIN COPY FOR INCOME TAX PURPOSES

ASSESSMENT SUBJECT TO LOCAL IMPROVEMENT LEVY
31,550

MUNICIPAL TAXES

Description			Assessment	Mill Rate	Levy
General Municipal			31,550	21.580	680.85
By-Law No.	**Term**	**Type**	**Frontage Levy**	**Mill Rate**	**Levy**
398	96	Sewer & Water	66.00	0.050	67.58
685/89	98	Sidewalk		2.440	76.98
762/93	98	Street		3.020	95.28

EDUCATION TAXES

Description	Assessment	Mill Rate	Levy
Provincial Education 1	31,550	7.960	251.14
Provincial Education 2	31,550	22.000	694.10

PROVINCIAL TAX CREDITS

(See Manitoba Enclosure for Additional Information)	Description	Credit
	Manitoba Resident Homeowner Tax Assistance	250.00

TOTAL TAXES DUE

Municipal Taxes	Education Taxes	Total Current Taxes	Prov. Credits	Net Taxes	*Arrears/Credits	Added Taxes	Total Taxes Due
920.69	945.24	1,865.93	(250.00)	1,615.93			1,615.93

Exercise 3: Foreign Exchange

Use the following chart for the foreign exchange questions that follow.

Buying Suggested Counter Rate	Quotations Expressed in Canadian Dollars Foreign Currency	Selling Suggested Counter Rate
1.3424	UNITED STATES .....DOLLARS	1.3784
2.0473	BANK OF ENGLAND.....POUND	2.1573
2.0473	SCOTLAND .....POUND	2.1573
2.0473	IRELAND NORTH .....POUND	2.1573
2.0839	IRELAND EIRE PUNT .....POUND	2.2172
1.0309	AUSTRALIA .....DOLLAR	1.1557
0.2189	DENMARK .....KRONER	0.2421
0.4245	E. CARIBBEAN.....DOLLAR	0.5870
0.2652	FINLAND .....MARKKA	0.3627
0.2507	FRANCE.....FRANC	0.2744
0.8572	GERMANY .....D. MARK	0.9375
0.1620	HONG KONG .....DOLLAR	0.1856
0.7644	NETHERLANDS.....GUILDER	0.8283
0.1975	NORWAY .....KRONER	0.2184

1. You are planning a trip to the United States. You estimate you will need $500.00 American dollars on the holiday. Use the foreign cash bulletin to calculate the cost of purchasing $500.00 U.S. dollars with Canadian money.
2. Your grandmother who lives in Hong Kong sends you a money order for $1000 Hong Kong dollars. Calculate the amount of Canadian money you would receive using the foreign cash bulletin.
3. The Yearbook Club ordered a manual (not available in Canada) from a publisher in the United States. The manual is priced at $21.95. What should the manual cost in Canadian dollars?
4. On a recent Travel Club trip to Finland a student purchased a sweatshirt for 200 markkas and placed the amount on the credit card bill. What amount will appear on the student's monthly credit card statement when the bank makes the conversion to Canadian dollars?
5. A science teacher ordered a piece of equipment needed to attempt a science project from a company in England. The price in the British catalogue was 10 pounds. What would the teacher pay in Canadian dollars for a money order for this amount?

Exercise 3: Foreign Exchange (continued)

6. On a recent trip to the United States you purchased $500 American dollars at your local bank. At the end of the trip you returned to the bank and wished to cash $200 American dollars back into Canadian dollars. The teller advised you that you might not want to do this if you were planning a second trip to the United States in the near future. Explain why this advice might be worthwhile. Support your argument with numerical calculations.

7. You are planning a trip to Disneyland. Your investigation finds that four nights in the local camping area will cost you $40.00 U.S. per night. In addition you will pay $100.00 to purchase a 3-day pass into Disneyland. What would this part of the trip cost you in Canadian dollars?

8. As a twenty-fifth wedding anniversary for your parents you and your siblings are thinking of sending them to Australia. Air-fare will be $2 500.00 Canadian per person. In addition, you estimate that it will cost $80.00 per night for 14 nights in Australian money for hotels and $40.00 per day for 14 days in Australian currency for meals. What would be your share of the total cost if there were four children in your family and you were paying in Canadian currency?

9. As an importer of goods into Canada you are faced with the following choice:

   Company A: Sells the product for 10 Scottish pounds each.

   Company B: Sells a similar product for 25 guilders (Netherlands's currency).

   Assuming the products are of equal quality, which product would you recommend on the basis of price alone?

10. Two friends were having a discussion about where they should buy their U.S. dollars for a trip to the Super Bowl. One friend said that it was not worthwhile to spend 35% to buy American money at a Canadian bank when he could go to a bank in the United States, where only 28% was deducted from his Canadian money when he converted to U.S. dollars. The second friend was a little skeptical of the argument presented by his friend and asked you to research this question: "Where would I get the better deal when purchasing American money — at a Canadian bank or at one in the United States?" Explain what advice you would give this individual supporting your conclusions with numerical calculations.

Exercise 4: Unit Costs

1. a) Milk was being sold in these sizes:
   — $3.89 for 4 litres
   — $2.69 for 2 litres
   — $1.65 for 1 litre
   Calculate the unit price per litre.
   b) Give reasons why you might purchase a smaller size of a product such as milk even though the larger size has a lower unit price.
2. Suppose you wished to purchase a laundry cleaner. Your options are as follows:
   a) regular laundry cleaner at $4.19 for 500 mL
   b) industrial strength laundry cleaner at $10.50 for 1 L
   Calculate each unit price.
   Why might someone buy the laundry cleaner with the higher unit price?
   Which laundry cleaner do you think is the better buy for your home?
3. Soft drink A was being sold in four different sizes on a particular day:
   — $1.69 for 2 litres
   — $0.99 for 1 litre
   — $5.99 for 24 cans with 355 mL in each can
   — $3.99 for 12 cans with 355 mL in each can
   a) Determine the unit cost per litre of each option.
   b) What do you consider when purchasing different-sized soft drinks? Are there any exceptions to your normal buying patterns?
   c) As shown in this example, unit costs can differ for a particular product. Why do you think that bottlers might price their products with such a discrepancy in unit costs?
4. When making comparisons supermarkets will often determine the price/100 g because the price/gram can be very small and, therefore, confusing to consumers.
   Brand A corn flakes are being sold in two different sizes:
   — $2.79 for a 525 g box
   — $3.19 for a 675 g box
   a) Determine the unit price for 100 grams.
   b) At the same time, Brand B corn flakes are being sold at $1.99 for 675 g. Determine the unit price per 100 g. Give reasons why people choose to purchase Brand A versus Brand B.
5. Brand A cookies are being sold in two different sizes:
   — $4.59 for a 600 g box
   — $2.99 for a 400 g box
   Determine the unit price for 100 grams.
   What can you conclude from your calculations in questions 4 and 5?

Exercise 4: Unit Costs (continued)

6. Recently, grocery shopping products are being sold in large sizes. What the large size is called depends upon the manufacturer or the store. In this particular supermarket the larger size is referred to as a Large Pack.

   Lean ground beef was being sold in two types of packages:

   — $3.60 for 0.585 kg in a smaller package

   — $9.48 for 2.160 kg in the Large Pack

   Determine the price per kilogram. Based on your result, what factors would you consider when determining your meat purchase at this store?

7. Another way to compare prices is price per 100 mL. Brand A contact lens solution was selling for $8.49 for 355 mL or $7.49 for 240 mL.

   Determine the unit price per 100 mL.

8. One of the most difficult types of products to compare is medicine sold over the counter. Consider the following two types of antacid tablets:

   Brand A is being sold in a box of 50 tablets. The recommended dosage is one tablet per time. The box of 50 tablets cost $12.96.

   Brand B is being sold in a box of 64 tablets. The recommended dosage is two tablets per time. The box of 64 tablets cost $8.96.

   a) Given the current pricing arrangement, which product appears to be the better deal? (Determine the price per dosage.)

   b) To increase sales, the manufacturers of Brand B decide to place a $3.00-off coupon on the box. To receive the discount, all you have to do is remove the sticker on the box and hand it to the cashier. Compare the price per dosage of the two products now that the coupon offer is in effect.

   c) What factors, other than price, would you use to determine which of the two products you would purchase?

   d) Discuss with your parents what impact coupons and sales have on their purchases.

9. Another type of product that is difficult to compare accurately is paper products. For example, paper towels are usually sold in different units:

   — Brand A rolls 2 Rolls 64 two-ply sheets per roll for $2.69

   — Brand B rolls 75 two-ply sheets per roll for $2.49

   — Brand C rolls 100 sheets two-ply sheets per roll for $1.79

   Prepare a written report investigating the various paper products available in your community. Explain what factors you would consider when testing the products from various companies.

   With a partner, conduct an experiment to determine whether the different products have equal absorbency or whether there appears to be a significant difference between brands.

   Which product are you prepared to recommend to your classmates?

Exercise 5: Chequebook Records

(See Attachment B-1 for blank chequebook records.)

1. Steve Lang has a personal chequing account with a beginning monthly balance of $1200.00. Using the following information, fill in a chequebook record to determine the end-of-month balance in Steve's personal chequing account:

   May 3: Wrote cheque 103 to Bright Dry Cleaners for $50.00 for dry-cleaning

   May 10: Wrote cheque 104 to ABC Grocery for $200.00 for food

   May 20: Wrote cheque 105 to J.W. Landlord for $600.00 for rent

   May 25 Made a deposit of $250.00 into the account

   May 28: Wrote cheque 106 to Minister of Finance for driver's license for $30.00

   May 30: Monthly bank service charge was $10.00

2. Jim Horner has a personal chequing account with a beginning monthly balance of $800.00. Using the following information, fill in a chequebook record to determine the end-of-month balance in Jim's personal chequing account:

   June 3: Wrote cheque 511 to Manitoba Coop for $150.00 for groceries.

   June 12: Wrote cheque 512 to VISA for $200.00 on account.

   June 20: Made a deposit of $400.00 into the account.

   June 26: Wrote cheque 513 to Wilson Properties for $500.00 for rent.

   June 30: Wrote cheque 514 to Minister of Finance for quarterly Autopac Insurance for $230.00.

   June 30: Monthly bank service charge $10.00.

   June 30: Wrote cheque 515 to MTS for the monthly telephone bill for $36.15

Exercise 6: Account Reconciliation

(See Attachment B-2 for blank reconciliation forms.)

Use an account reconciliation form to answer the following questions.

1. Assume the account statement dated June 30, 19--, prepared for yourself shows a final balance of $596.12.

   When you examine your chequebook record you discover that the balance you have written down is $1765.42. After a careful examination of the account records and your own records, you discover the following information:

   a) The following cheques were outstanding: no. 45, $35.80; no. 47, $76.10; no. 50, $225.80.

   b) Deposit placed in the night depository on June 30, 19-- was for $1485.00.

   c) The account statement showed a debit balance of $12 for the rental of a small safety deposit box.

   d) Account service charges for the month of June were $10.

   Prepare an account reconciliation statement given the above information (don't forget to subtract all account charges from your chequebook total).

2. Assume the account statement dated May 31, 19--, prepared for yourself shows a final balance of $840.00.

   When you examine your chequebook record you discover that the balance you have written down is $944.00. After a careful examination of the account records and your own records, you discover the following information:

   a) The following cheques were outstanding: no. 160, $42.00; no. 167, $49.00; no. 183, $120.00.

   b) Deposit placed in the night depository on May 31, 19-- was for $300.00.

   c) Account service charges for the month of June were $15.00.

   Prepare an account reconciliation statement given the above information (don't forget to subtract all account charges from your chequebook total).

Exercise 6: Account Reconciliation (continued)

3. Reconcile the account statement information with the chequebook balance found below. Use an account reconciliation form.

J R. Student
Box 126
Orleans, MB

Credit Union of Manitoba

Acct # 55555

DATE	DESCRIPTION	DEBITS	CREDITS	BALANCE
May 1	Balance Forward			127.18
May 1	Deposit		520.15	647.33
May 3	Cheque 346	425.00		222.33
May 8	Cheque 347	57.66		164.67
May 10	Deposit		80.89	245.56
May 13	Cheque 348	42.38		203.18
May 14	Cheque 350	103.56		99.62
May 15	Deposit		420.15	519.77
May 19	Cheque 349	144.34		375.43
May 23	Cheque 351	125.00		250.43
May 28	Cheque 353	36.15		214.28
May 31	Service Charge	4.75		209.53

DATE	CHEQUE NO	CHEQUES ISSUED TO OR DESCRIPTION OF DEPOSIT	CHEQUE AMOUNT	✓	DEPOSIT AMOUNT	DEDUCT CHEQUES ADD DEPOSITS	BALANCE FORWARD
							127 18
May 1		TO Deposit			520 15	CHEQUE – DEPOSIT +	520 15
		FOR				BALANCE →	647 33
1	346	TO Ace Holdings	425 00			CHEQUE – DEPOSIT +	425 00
		FOR Rent				BALANCE →	222 33
6	347	TO Pierre & Mario's	57 00			CHEQUE – DEPOSIT +	57 66
		FOR Groceries				BALANCE →	164 67
10	348	TO Manitoba Hydro	42 00			CHEQUE – DEPOSIT +	42 38
		FOR Electrical Bill				BALANCE →	122 29
10		TO Deposit			80 89	CHEQUE – DEPOSIT +	80 89
		FOR				BALANCE →	203 18
12	349	TO Bill's Auto Repair	144 34			CHEQUE – DEPOSIT +	144 34
		FOR Car				BALANCE →	58 84
12		TO Deposit			420 15	CHEQUE – DEPOSIT +	420 15
		FOR				BALANCE →	478 99
15	350	TO Hubert's Gen. Store	103 56			CHEQUE – DEPOSIT +	103 56
		FOR Clothes				BALANCE →	375 43
18	351	TO Dr. Howie	125 00			CHEQUE – DEPOSIT +	125 00
		FOR Dentist				BALANCE →	250 43
20	352	TO Active Sporting Goods	17 86			CHEQUE – DEPOSIT +	17 86
		FOR Golf Glove				BALANCE →	232 57
25	353	TO T.C.I.	36 15			CHEQUE – DEPOSIT +	36 15
		FOR Sweatsuit				BALANCE →	196 42
31	354	TO Wilson's Store	54 76			CHEQUE – DEPOSIT +	54 76
		FOR Groceries				BALANCE →	141 66
31		TO Deposit			45 00	CHEQUE – DEPOSIT +	45 00
		FOR				BALANCE →	186 66

Exercise 6: Account Reconciliation (continued)

4. Reconcile the account statement information with the chequebook balance found below. Use an account reconciliation form.

J. Doe
Box 186
Anywhere, MB

Bank of Manitoba

Acct # 33333

DATE	DESCRIPTION	DEBITS	CREDITS	BALANCE
June 1	Balance Forward			151.63
June 3	Deposit		85.16	236.79
June 6	Cheque 125	11.62		225.17
June 13	Cheque 126	16.33		208.84
June 18	Cheque 127	37.40		171.49
June 18	Deposit		105.90	277.34
June 19	Cheque 128	25.61		251.73
June 26	Cheque 129	42.07		209.66
June 28	Cheque 120	18.82		190.84
June 30	Cheque 131	35.00		155.84
June 30	Service Charge	6.60		149.24

DATE	CHEQUE NO	CHEQUES ISSUED TO OR DESCRIPTION OF DEPOSIT	CHEQUE AMOUNT		✓	DEPOSIT AMOUNT		DEDUCT CHEQUES ADD DEPOSITS	BALANCE FORWARD	
									151	63
June 1	125	TO Dolly's Flowers	11	62				CHEQUE – DEPOSIT +	11	62
		FOR Corsage						BALANCE →	140	01
3		TO Deposit				85	16	CHEQUE – DEPOSIT +	85	16
		FOR						BALANCE →	225	17
7	126	TO Tate's Dry Cleaners	16	33				CHEQUE – DEPOSIT +	16	33
		FOR Shirts						BALANCE →	208	84
12	127	TO Coop	37	40				CHEQUE – DEPOSIT +	37	40
		FOR Groceries						BALANCE →	171	44
12	128	TO Fashion Fabrics	25	61		80	89	CHEQUE – DEPOSIT +	25	61
		FOR Birthday Gift						BALANCE →	145	83
18		TO Deposit				105	90	CHEQUE – DEPOSIT +	105	90
		FOR						BALANCE →	251	73
21	129	TO R & R Repair	42	07				CHEQUE – DEPOSIT +	42	07
		FOR Car Repair						BALANCE →	209	66
23	130	TO Bill's Hardware	18	82				CHEQUE – DEPOSIT +	18	82
		FOR Electric Cord						BALANCE →	190	84
24		TO Deposit				27	70	CHEQUE – DEPOSIT +	27	70
		FOR						BALANCE →	218	54
24	131	TO Select Seat	35	00				CHEQUE – DEPOSIT +	35	00
		FOR Concert						BALANCE →	183	54
27	132	TO Times	9	98				CHEQUE – DEPOSIT +	9	98
		FOR Newspaper						BALANCE →	173	56
30		TO Deposit				12	30	CHEQUE – DEPOSIT +	12	30
		FOR						BALANCE →	185	86
		TO						CHEQUE – DEPOSIT +		
		FOR						BALANCE →		

Attachment B-1

DATE	CHEQUE NO.	CHEQUES ISSUED TO OR DESCRIPTION OF DEPOSIT	CHEQUE AMOUNT		✓	DEPOSIT AMOUNT		DEDUCT CHEQUES ADD DEPOSITS	BALANCE FORWARD	
		TO						CHEQUE – DEPOSIT +		
		FOR						BALANCE ➜		
		TO						CHEQUE – DEPOSIT +		
		FOR						BALANCE ➜		
		TO						CHEQUE – DEPOSIT +		
		FOR						BALANCE ➜		
		TO						CHEQUE – DEPOSIT +		
		FOR						BALANCE ➜		
		TO						CHEQUE – DEPOSIT +		
		FOR						BALANCE ➜		
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		TO						CHEQUE – DEPOSIT +		
		FOR						BALANCE ➜		
		TO						CHEQUE – DEPOSIT +		
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		TO						CHEQUE – DEPOSIT +		
		FOR						BALANCE ➜		
		TO						CHEQUE – DEPOSIT +		
		FOR						BALANCE ➜		
		TO						CHEQUE – DEPOSIT +		
		FOR						BALANCE ➜		
		TO						CHEQUE – DEPOSIT +		
		FOR						BALANCE ➜		

Attachment B-2

STATEMENT OF RECONCILIATION	
FINAL BALANCE shown on this account statement	FINAL BALANCE shown on this cheque register
ADD DEPOSITS made since statement date.	ADD ALL WITHDRAWALS and CHEQUES
TOTAL:	**TOTAL:**

STATEMENT OF RECONCILIATION	
FINAL BALANCE shown on this account statement	FINAL BALANCE shown on this cheque register
ADD DEPOSITS made since statement date.	ADD ALL WITHDRAWALS and CHEQUES
TOTAL:	**TOTAL:**

Unit B
Personal Finance
Solutions

Exercise 1: Income — Solutions

1. a) Regular Pay: 40 x 10.20 = $408.00
   Overtime Pay: 5.5 x 10.20 x 1.5 = 84.15
   Gross Pay $492.15

   b) Regular Pay: 40 x 14.15 = $566.00
   Overtime Pay: 4 x 14.15 x 1.5 = 84.90
   Gross Pay $650.90

   c) Regular Pay: 40 x 20.00 = $800.00
   Overtime Pay: 6 x 20.00 x 1.5 = 180.00
   Gross Pay $980.00

2 a) Regular Hours: 8 + 8 + 7 + 8 + 7.5 = 38.5
   Overtime Hours: 1 + 0.5 + 0 + 2 + 0 = 3.5

   Regular Pay: 38.5 x 8.20 = $315.70
   Overtime Pay: 3.5 x 8.20 x 1.5 = 43.05
   Gross Pay $358.75

   b) Regular Hours: 8 + 8 + 8 + 8 + 8 = 40
   Overtime Hours: 0 + 1.5 + 2 + 1 + 0.5 = 5

   Regular Pay: 40 x 12.25 = $490.00
   Overtime Pay: 5 x 12.25 x 1.5 = 91.88
   Gross Pay $581.88

   c) Regular Hours: 8 + 8 + 8 + 8 + 8 = 40
   Overtime Hours: 2 + 0 + 1 + 3 + 1.5 = 7.5

   Regular Pay: 40 x 18.00 = $720.00
   Overtime Pay: 7.5 x 18.00 x 1.5 = 202.50
   Gross Pay $922.50

Teacher Note: The remaining answers demonstrate the calculation of gross pay. Use the current tables for the year in which you are teaching this course to determine net pay.

3. Regular Pay 40 x 14.00 = $560.00
   Overtime Pay 6 x 14.00 x 1.5 = 126.00
   Double Pay 8 x 14.00 x 2 = 224.00
   Gross Pay $910.00

4. 8% commission $1000 x 8% = $ 80.00
   15% commission $4000 x 15% = 600.00
   Gross Pay $680.00

5. Sales Subject to Commission: $3150 – $900 = $2250.00
   Commission: $2250 x 5% = $112.50
   Total Earnings: $250.00 + $112.50 = $362.50

Exercise 1: Income — Solutions (continued)

6. Regular Pay: 40 x 5.75 = $230.00
   Overtime Pay: 5 x 5.75 x 1.5 = 43.13
   Tips 185.00
   Gross Pay $458.13

Teacher Note: Many people do not realize that tips are considered to be part of income and should be reported on their income tax return. When calculating income tax, CPP, and EI in the above question, the employer would use the $230.00 + $43.13 = $273.13 amount because this is the amount of income the employer is required to report from the company's point of view.

7. 1st Commission: $12 000 x 5% = $600.00
   2nd Commission: $12 000 x 7% = 840.00
   Monthly Salary: 700.00
   Gross Pay $2140.00

8. Company A:
   Regular Pay: 40 x 16.00 = $640.00
   Overtime Pay: 12 x 16.00 x 1.5 = 288.00
   Gross Pay: $928.00

   Company B:
   Regular Hours: 8 + 7 + 8 + 8 + 8 = 39
   Overtime Hours: 3 + 0 + 3 + 4 + 3 = 13

   Regular Pay: 39 x 16.00 = $624.00
   Overtime Pay: 13 x 16.00 x 1.5 = 312.00
   Gross Pay: $936.00

   Therefore, Company B is better for this particular week by $936.00 – $928.00 = $8.00

   Teacher Note: Point out that normally people would prefer to be paid at the overtime rate rather than the regular rate. This is why Company B is a better plan if you are not likely to work a full 8 hour day. (For example, construction workers who are rained out for a day receive no overtime pay even though they may have worked a 12-hour day the day before).

9. Gross Pay: 100 x $1.20 x 5 days = $600.00

10. Regular Pay: $16.00 x 40 = $640.00
    Overtime: $16.00 x 12 x 1.5 = 288.00
    Gross Pay: $928.00

Exercise 2: Property Tax — Solutions

1. \$90 000 x 45% = \$40 500

   $40\,500 \times \frac{62}{1000} = \$2511.00$

   Total Bill: \$2511.00 + 180.00 = \$2691.00

2. Original Tax: \$80 000 x 45% = 36 000 x $\frac{55}{1000}$ = \$1980.00

   New Tax: \$90 000 x 45% = 40 500 x $\frac{55}{1000}$ = \$2227.50

   Reassessment Cost: \$2227.50 – \$1980.00 = \$247.50

3. Assessed Value: \$85 000 x 45% = \$38 250

   General Tax: \$38 250 x $\frac{70}{1000}$ = \$2677.50

   Sewer: 15 m x \$3.87 = \$58.05
   Sidewalks: 15 m x \$2.50 = \$37.50
   Total Bill: \$2677.50 + 58.05 + 37.50 = \$2773.05

4. a) $\frac{69\,000\,000}{780\,000\,000} \times 1000 = 88 \text{ mills}$

   b) $\frac{82\,000\,000}{852\,000\,000} \times 1000 = 96 \text{ mills}$

   c) $\frac{95\,000\,000}{945\,000\,000} \times 1000 = 101 \text{ mills}$

Exercise 2: Property Tax — Solutions (continued)

5.

STATEMENT AND DEMAND FOR TAXES

REAL PROPERTY **ROLL NUMBER:**
PAGE: 1 OF 1

PROPERTY DESCRIPTION

ROLL NUMBER	WARD	Lot/Section	Blk/Twp	Plan/Range	Frontage/Area	Dwell. Units
4120		3	6	120	65.2 F	1

Civic Address:

Title or Deed No.	Current Assessment Land	Current Assessment Buildings	Status Code	Total Assessment	Prop. Class	Portion %	Total Port. Assessment
B630	9200	65 000	T	74 200	10	45	

- ERRORS AND OMISSONS EXCEPTED
- ALL LAND IN ARREARS FOR MORE THAN ONE YEAR SHALL BE SOLD FOR TAXES
- ALL CHEQUES MUST BE MADE PAYABLE IN CANADIAN FUNDS
- CASH REGISTER RECEIPT IMPRESSION OR BANK RECEIPT CONSTITUTES OFFICIAL RECEIPT
- RETAIN COPY FOR INCOME TAX PURPOSES

ASSESSMENT SUBJECT TO LOCAL IMPROVEMENT LEVY
33 930

MUNICIPAL TAXES

Description			Assessment	Mill Rate	Levy
General Municipal			33 930	22.5	751.28
By-Law No.	**Term**	**Type**	**Frontage Levy**	**Mill Rate**	**Levy**
412	98	Sewer & Water	70.00	0.050	71.67
506	99	Sidewalk		2.460	82.14
508	99	Street		3.150	105.18

EDUCATION TAXES

Description	Assessment	Mill Rate	Levy
Provincial Education 1	33 390	8.1	270.46
Provincial Education 2	33 390	21.5	717.89

PROVINCIAL TAX CREDITS

(See Manitoba Enclosure for Additional Information)	Description	Credit
	Manitoba Resident Homeowner Tax Assistance	250.00

TOTAL TAXES DUE

Municipal Taxes	Education Taxes	Total Current Taxes	Prov. Credits	Net Taxes	*Arrears/Credits	Added Taxes	Total Taxes Due
1010.27	988.35	1998.62	−250.00	1748.62	ϕ	ϕ	1748.62

Exercise 2: Property Tax — Solutions (continued)

6.

Property Tax Template

Land	Buildings	Assessed Value	Class	%	Assess	Total Port. Assessment
8900.00	61 200.00	70 100.00	10.00	45.00	31 545.00	31 550.00
Municipal Taxes		General Municipal		31 550.00	21.580	680.85
		Sewer & Water		66.00	0.050	67.58
		Sidewalk			2.440	76.98
		Street			3.020	95.28
Education Taxes		Provincial Education 1		31 550.00	7.960	251.14
		Provincial Education 2		31 550.00	22.000	694.10

Total Taxes Due							
Municipal Taxes	Educat. Taxes	Total Taxes	Prov. Credits	Net Taxes	Arrears/ Credits	Added Taxes	Taxes Due
920.69	945.24	1865.93	–250.00	1615.93			1615.93

Note that in this municipality the assessed value is rounded to the nearest 10 dollars. When preparing a spreadsheet, students will need to use a round function.

For example, using Lotus 1-2-3 the round function is written as: @round(f6, –1).

Exercise 2: Property Tax — Solutions (continued)

6. i)

Property Tax Template

Land	Buildings	Assessed Value	Class	%	Assess	Total Port. Assessment
8900.00	61 200.00	70 100.00	10.00	45.00	31 545.00	31 550.00
Municipal Taxes		General Municipal		31 550.00	25.000	788.75
		Sewer & Water		66.00	0.050	67.58
		Sidewalk			2.440	76.98
		Street			3.020	95.28
Education Taxes		Provincial Education 1		31 550.00	7.960	251.14
		Provincial Education 2		31 550.00	22.000	694.10
Provincial Tax Credits		Manitoba Resident Homeowner Tax Assistance				–250.00

Total Taxes Due							
Municipal Taxes	Educat. Taxes	Total Taxes	Prov. Credits	Net Taxes	Arrears/ Credits	Added Taxes	Taxes Due
1028.59	945.24	1973.83	–250.00	1723.83			1723.83

Exercise 2: Property Tax — Solutions (continued)

6. ii)

Property Tax Template

Land	Buildings	Assessed Value	Class	%	Assess	Total Port. Assessment
8900.00	69 000.00	77 900.00	10.00	45.00	35 055.00	35 060.00
Municipal Taxes		General Municipal		35 060.00	21.580	756.59
		Sewer & Water		66.00	0.050	67.75
		Sidewalk			2.440	85.55
		Street			3.020	105.88
Education Taxes		Provincial Education 1		35 060.00	7.960	279.08
		Provincial Education 2		35 060.00	22.000	771.32
Provincial Tax Credits		Manitoba Resident Homeowner Tax Assistance				–250.00

Total Taxes Due							
Municipal Taxes	Educat. Taxes	Total Taxes	Prov. Credits	Net Taxes	Arrears/ Credits	Added Taxes	Taxes Due
1015.77	1050.40	2066.17	–250.00	1816.17			1816.17

Notes:
1. Remember assessed value is rounded to the nearest 10 dollars.
2. Reassessment resulted in an increase in property taxes of $1816.17 – $1615.93 = $200.24

Exercise 2: Property Tax — Solutions (continued)

6. iii)

Property Tax Template

Land	Buildings	Assessed Value	Class	%	Assess	Total Port. Assessment
8900.00	61 200.00	70 100.00	10.00	45.00	31 545.00	31 550.00
Municipal Taxes		General Municipal		31 550.00	21.580	680.85
		Sewer & Water		66.00	0.050	67.75
		Sidewalk			2.440	76.98
		Street			3.020	95.28
Education Taxes		Provincial Education 1		31 550.00	7.960	251.14
		Provincial Education 2		31 550.00	26.000	820.30
Provincial Tax Credits		Manitoba Resident Homeowner Tax Assistance				–250.00

Total Taxes Due							
Municipal Taxes	Educat. Taxes	Total Taxes	Prov. Credits	Net Taxes	Arrears/ Credits	Added Taxes	Taxes Due
920.69	1071.44	1992.13	–250.00	1742.13			1742.13

The school board decision resulted in an increase in taxes of $1742.13 – $1615.93 = $126.20.

Exercise 2: Property Tax — Solutions (continued)

iv) Property Tax Template

Land	Buildings	Assess	Class	%	Assess	Total Port. Assessment
8900.00	61 200.00	70 100.00	10.00	45.00	31 545.00	31 550.00
Municipal Taxes		General Municipal		31 550.00	21.580	680.85
		Sewer & Water		66.00	0.050	67.75
		Sidewalk			2.440	76.98
		Street			3.020	95.28
		Roads			3.050	96.23
Education Taxes		Provincial Education 1		31 550.00	7.960	251.14
		Provincial Education 2		31 550.00	26.000	820.30
Provincial Tax Credits		Manitoba Resident Homeowner Tax Assistance				–250.00

Total Taxes Due							
Municipal Taxes	Educat. Taxes	Total Taxes	Prov. Credits	Net Taxes	Arrears/ Credits	Added Taxes	Taxes Due
1016.92	945.24	1962.16	–250.00	1712.16			1712.16

The new by-law raised taxes by $1712.16 – $1615.93 = $96.23.

Exercise 2: Property Tax — Solutions (continued)

6. v)

Property Tax Template

Land	Buildings	Assessed Value	Class	%	Assess	Total Port. Assessment
8900.00	61 200.00	70 100.00	10.00	45.00	31 545.00	31 550.00
Municipal Taxes		General Municipal		31 550.00	22.580	712.40
		Sewer & Water		66.00	0.050	67.75
		Sidewalk			2.440	76.98
		Street			3.020	95.28
Education Taxes		Provincial Education 1		31 550.00	7.960	251.14
		Provincial Education 2		31 550.00	26.000	694.10
Provincial Tax Credits		Manitoba Resident Homeowner Tax Assistance				–250.00

Total Taxes Due							
Municipal Taxes	Educat. Taxes	Total Taxes	Prov. Credits	Net Taxes	Arrears/ Credits	Added Taxes	Taxes Due
952.24	945.24	1897.48	–250.00	1647.48			1647.48

The one mill increase raised taxes by $1647.48 – $1615.93 = $31.55.

Exercise 3: Foreign Exchange — Solutions

1. $500.00 x 1.3784 = $689.20 Canadian dollars

2. $1000.00 x 0.1620 = $162.00 Canadian dollars

3. $21.95 x 1.3784 = $30.26 Canadian dollars

4. 200 markkas x 0.3627 = $72.54 Canadian dollars

   Point out that using your credit card in a foreign country is the same as purchasing the foreign money from your bank. This means the bank is selling you the money.

5. 10 pounds x 2.1573 = $21.57 Canadian dollars

6. If you convert the American money to Canadian money, the bank is buying the money from you. They will give you $200 x 1.3424 = $268.48 Canadian dollars.

   In the future, when you purchase U.S. dollars, the bank is selling you the money. They will charge you $200 x 1.3784 = $275.68 Canadian dollars.

   This transaction costs you $275.68 – $268.48 = $7.20.

   Therefore, you probably should hold onto the American dollars unless your next trip will be in the far future.

7. Camping $40.00 x 4 = $160.00
   Disneyland 100.00
   $260.00 U.S. dollars
   Cost $260.00 x 1.3784 = $358.38 Canadian dollars

8. Air Fare $2500.00 x 2 = $5000.00 Canadian dollars
   Hotel: $80.00 x 14 x 1.1557 = $1294.38 Canadian dollars
   Meals: $40.00 x 14 x 1.1557 = $ 647.19 Canadian dollars
   $6941.57 Canadian dollars

   Your share would be $\frac{\$6941.57}{4} = \$1735.39$

9. Company A: 10 pounds x 2.1573 = $21.57
   Company B: 25 guilders x 0.8283 = $20.71
   Therefore, if price is your only consideration then Company B is better by
   $21.57 – $20.71 = $0.86

Exercise 3: Foreign Exchange — Solutions (continued)

10. Suppose you wished to purchase $2000.00 for the trip. At a Canadian bank this would cost you

$$\$2000.00 \times 1.35 = \$2700.00$$

The question that has to be answered is: How many American dollars would you get for the same $2700.00 Canadian?

When you go to the U.S. bank they will apply the 28% charge on your Canadian funds.

$$\$2700.00 \times 28\% = \$756.00 \text{ charge}$$

You will then receive: $2700.00 – $756.00 = $1944.00 U.S. dollars.

As demonstrated this transaction cost you:

$$\$2000.00 - \$1944.00 = \$56.00 \text{ U.S. dollars}$$

When discussing this question at a bank the banker advised that in almost all cases a currency is more valuable in the country of origin. That is, the Canadian dollar is worth more in Canada against the American dollar.

Unfortunately, many people see only the percent rates given and not the dollar amounts involved in the calculation.

Ask students to bring this question home to their parents and see what percent of them believe the 28% rate is a better deal.

Exercise 4: Unit Costs — Solutions

1. a) $3.89 ÷ 4 = $0.9725 per litre
   $2.69 ÷ 2 = $1.345 per litre
   $1.65 ÷ 1 = $ 1.65 per litre

   b) Discuss the possibility of spoilage if you do not use up the larger size before the product spoils.

2. Using a unit price per litre we get:
   Regular: $4.19 x 2 = $8.38 per litre
   Industrial: $10.50 per litre

   Discuss the choice of a product with a higher unit cost that might be a better product. Emphasize to students that not all of these products do what they are advertised to do, and may in fact, be an unwise purchase. Students should be aware of consumer reports that may assist them in making wise choices.

3. $1.69 ÷ 2 = $0.845 per litre
   $0.99 ÷ 1 = $0.99 per litre

   Making price comparisons with cans is a little more challenging for the average consumer. The first step is to calculate the total litres in each case.

   355 mL x 24 = 8520 mL = 8.52 L

   355 mL x 12 = 4260 mL = 4.26 L

   Now calculate the price per litre.

   $5.99 ÷ 8.52 = $0.703 per litre

   $3.99 ÷ 4.26 = $0.937 per litre

   As you can see, on this particular day the 24 cans for $5.99 is the better buy. On observing people making their choices it was obvious that most people were not examining the unit cost and a number of people purchased the 12 cans even though it was not the best deal. Emphasize that with products such as this the best price can change from purchase to purchase.

4. a) $2.79 ÷ 525 x 100 = $0.531 per 100 g
   $3.19 ÷ 675 x 100 = $0.473 per 100 g

   b) $1.99 ÷ 675 x 100 = $0.295 per 100 g

   Many people choose the name brand because of factors such as advertising or the fact their families have always purchased this particular product. As well, many people have either never tried the brand B product or they may have tried it and not liked the product. Discuss the idea of having an open mind when experimenting with different products and companies prior to making final choices.

5. $4.59 ÷ 600 x 100 = $0.765 per 100 g
   $2.99 ÷ 400 x 100 = $0.748 per 100 g

   Students should draw the conclusion that although the largest size is often the best deal, this is not always the case.

Exercise 4: Unit Costs — Solutions (continued)

6. $3.60 ÷ 0.585 = $6.154 per kg
   $9.48 ÷ 2.160 = $4.389 per kg

   Factors such as family size and whether the product can safely be stored until used must be considered when making this type of purchase.

7. $8.49 ÷ 355 x 100 = $2.392 per 100 mL
   $7.49 ÷ 240 x 100 = $3.121 per 100 mL

   Emphasize the significant difference in these prices and advise students that this discrepancy occurs with a lot of different products.

8. a) Product A: $12.96 ÷ 50 = $0.2592 per dosage
      Product B: $8.96 ÷ 32 = $0.28 per dosage

      Note that Brand B requires two tablets per dosage. Given this pricing arrangement, Brand A is the better deal based on price alone.

   b) With the discount coupon the new price per dosage of Brand B would be:
      Brand B: $5.96 ÷ 32 = $0.1863 per dosage.
      With the coupon offer Brand B now has a lower price per dosage.

   c) Most consumers taking medication have a personal preference. Students should realize that medication has different side effects on people and that consumers often make their purchases based on their past experiences with a particular product.

9. Answers will vary.

Exercise 5: Chequebook Record — Solutions

1.

DATE	CHEQUE NO.	CHEQUES ISSUED TO OR DESCRIPTION OF DEPOSIT	CHEQUE AMOUNT	✓	DEPOSIT AMOUNT	DEDUCT CHEQUES ADD DEPOSITS	BALANCE FORWARD
							1200 00
May 3		TO Bright Dry Cleaners	50 00			CHEQUE – DEPOSIT +	50 00
		FOR Dry Cleaning				BALANCE →	1150 00
10		TO ABC	200 00			CHEQUE – DEPOSIT +	200 00
		FOR Food				BALANCE →	950 00
20		TO J.W. Landlord	600 00			CHEQUE – DEPOSIT +	600 00
		FOR Rent				BALANCE →	350 00
25		TO Deposit			250 00	CHEQUE – DEPOSIT +	250 00
		FOR				BALANCE →	600 00
28		TO Minister of Finance	30 00			CHEQUE – DEPOSIT +	30 00
		FOR Driver's Licence				BALANCE →	570 00
30		TO Bank	10 00			CHEQUE – DEPOSIT +	10 00
		FOR Service Charge				BALANCE →	560 00

2.

DATE	CHEQUE NO.	CHEQUES ISSUED TO OR DESCRIPTION OF DEPOSIT	CHEQUE AMOUNT	✓	DEPOSIT AMOUNT	DEDUCT CHEQUES ADD DEPOSITS	BALANCE FORWARD
							800 00
June 3		TO Manitoba Coop	150 00			CHEQUE – DEPOSIT +	150 00
		FOR Groceries				BALANCE →	650 00
12		TO VISA	200 00			CHEQUE – DEPOSIT +	200 00
		FOR On Account				BALANCE →	450 00
20		TO Deposit	600 00		400 00	CHEQUE – DEPOSIT +	400 00
		FOR				BALANCE →	850 00
26		TO Wilson PRoperties	500 00			CHEQUE – DEPOSIT +	500 00
		FOR Rent				BALANCE →	350 00
30		TO Minister of Finance	230 00			CHEQUE – DEPOSIT +	230 00
		FOR Autopac				BALANCE →	120 00
30		TO Bank	10 00			CHEQUE – DEPOSIT +	10 00
		FOR Service Charge				BALANCE →	110 00
30		TO MTS	36 15			CHEQUE – DEPOSIT +	36 15
		FOR Telephone Bill				BALANCE →	73 85

Exercise 6: Account Reconciliation — Solutions

1. Step 1 is to subtract account charges in your chequebook record.

Original Balance in Chequebook		$1765.42
Less: Safety Deposit Box Charge	$12.00	
Account Service Charge	$10.00	22.00
Adjusted Chequebook Balance		$1743.42

Step 2 is to use a reconciliation form to get the reconciled amount:

Final Balance Shown on This Statement	Balance Shown in Chequebook Record
$596.12	$1743.42
Add Deposits Made Since Last Date on This Statement	**Add Cheques Issued But Not On Statement**
$1485.00	$35.80
	76.10
	225.80
TOTAL	TOTAL
$2081.12	$2081.12

THESE TOTALS SHOULD AGREE

2. Step 1 is to subtract account charges in your chequebook record.

Original Balance in Chequebook	$ 944.00
Less: Account Service Charge	15.00
Adjusted Chequebook Balance	$ 929.00

Step 2 is to use a reconciliation form to get the reconciled amount:

Final Balance Shown on This Statement	Balance Shown in Chequebook Record
$840.00	$ 929.00
Add Deposits Made Since Last Date on This Statement	**Add Cheques Issued But Not on Statement**
$ 300.00	$ 42.00
	49.00
	120.00
TOTAL	TOTAL
$1140.00	$1140.00

THESE TOTALS SHOULD AGREE

Exercise 6: Account Reconciliation — Solutions (continued)

3. Step 1 is to subtract account charges in your chequebook record.

Original Balance in Chequebook	$186.66
Less: Account Service Charge	4.75
Adjusted Chequebook Balance	$181.91

Step 2 is to use a reconciliation form to get the reconciled amount:

Final Balance Shown On This Statement	Balance Shown in Chequebook Record
$209.53	$181.91
Add Deposits Made Since Last Date on This Statement	**Add Cheques Issued But Not on Statement**
$45.00	$17.86
	54.76
TOTAL	TOTAL
$254.53	$254.53

THESE TOTALS SHOULD AGREE

4. Step 1 is to subtract account charges in your chequebook record.

Original Balance in Chequebook	$ 185.86
Less: Account Service Charge	6.60
Adjusted Chequebook Balance	$ 179.26

Step 2 is to use a reconciliation form to get the reconciled amount:

Final Balance Shown on This Statement	Balance Shown in Chequebook Record
$149.24	$179.26
Add Deposits Made Since Last Date on This Statement	**Add Cheques Issued But Not on Statement**
$27.70	$9.98
$12.30	
TOTAL	TOTAL
$189.24	$189.24

THESE TOTALS SHOULD AGREE

Unit C
Systems of Linear Equations

Teachers are advised to refer to *Senior 3 Applied Mathematics: A Foundation for Implementation* as their ***primary*** source of suggestions for instruction and assessment. The following exercises are an excellent resource if supplementary material is required.

Exercise 1: Solving Systems of Linear Equations

1. Plot the graphs of the following systems. Name the type of system graphed. Find the point(s) of intersection, if any, of each of the systems.

a) $y = 9x - 6$
$y = 3x$

b) $2y = x + 12$
$y + 3x = -1$

c) $y = -4x + 3$
$2y = -8x + 12$

d) $2x + y = 1$
$x + 2y = 5$

e) $y = 2(x + 3) - 4$
$y - 2x = 2$

f) $2y = x + 3$
$y = x + 4$

2. With the use of a graphing tool,
— graph the following linear equations
— find the y-intercept
— find the x-intercept

a) $y = x$

b) $y = -2x + 4$

c) $y = 15x + 4$

d) $y = \frac{1}{3}x + 1$

e) $y = 3x - \frac{1}{4}$

f) $y = \frac{-2}{5}x + 30$

g) $2y = x - 8$

h) $3x + y - 15 = 0$

3. Solve the following systems using a graphing tool.

a) $y = x$
$y + x = 10$

b) $2x + y = 30$
$y = x + 8.5$

c) $3x + 2y = 5$
$x - 3y = 12$

d) $2x + 4y = 8$
$2y = -x - 10$

e) $4y - 16x = 20$
$y = 4x + 5$

f) $2y = 10x - 100$
$y = 20x + 80$

g) $5x - y + 3 = 0$
$2y = 10x + 12$

h) $3x + y = 4$
$2y = -6x + 8$

i) $8x - 6y = -72$
$2(2x - 1) + 3(2y + 1) = 37$

4. Your friend missed yesterday's math class and, therefore, was not there when graphing systems of equations was discussed. Write the steps for him so that he can solve systems of equations using a graphing calculator. Ask someone who does not know the procedure to try your steps. Where does that person have difficulty following the procedure? Make appropriate alterations so that the steps are clear and easily understood.

Exercise 1: Solving Systems of Linear Equations (continued)

5. Solve the following systems where possible, using the substitution method. Use a graphing tool to verify your answers.

a) $x + y = 10$
$x = y$

b) $x = 5$
$2y - 3x = -11$

c) $y = x + 1$
$x + y = 7$

d) $x + y = -8$
$x - y = -12$

e) $x + 3y = 3$
$x - 6y = -6$

f) $2x + 4y = 8$
$3x + y = -13$

g) $6x + y = -15$
$x = y + 1$

h) $6x = 9y - 21$
$6y = 4x + 14$

i) $2(x + 3) + y = -3$
$2x + 2y = -12$

j) $x + y = 4(y - 2)$
$x - y = 2(y + 4)$

k) $\frac{x}{4} + y = 4$
$x + y = 31$

l) $0.05x + 0.01y = -1.8$
$0.03x - 0.02y = -1.6$

6. Solve the following systems where possible, using the addition or subtraction method. Use a graphing tool to verify your answers.

a) $x + y = -5$
$x = y = 1$

b) $2x + 2y = 8$
$3x + 2y = 10$

c) $2x + 3y = 3$
$3x - 2y = 11$

d) $2x + 3y = 1$
$4x - y = 2$

e) $0.01x + 0.02y = 0.04$
$2x + 3y = 16$

f) $2x + 5y = 13$
$3x + 2y = 3$

g) $2x + y = 4.5$
$3x - 2y = 2$

h) $2x + 7y = 9$
$0.5x + 2y = 3$

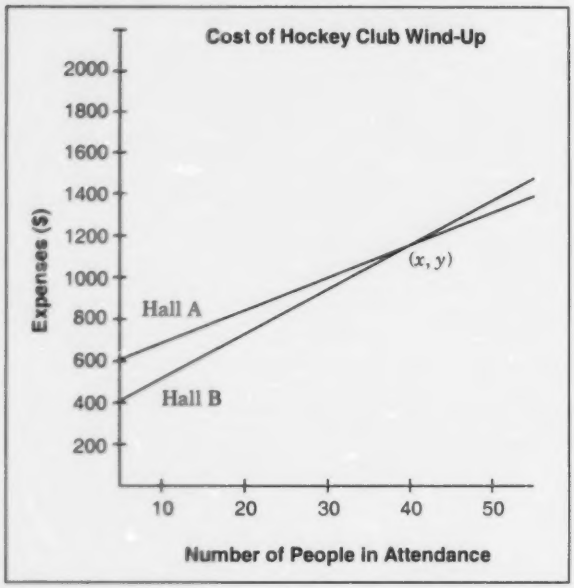
i) $2(x + 3) + 3(y - 1) = 3$
$4x + y = 0$

Exercise 2: Solving Problems

1. Create a system of equations whose solution is $\left(3, \frac{1}{2}\right)$.

2. Which algebraic method would you use to solve the following system: $3x + 2y = 15$, $5x - 3y = 20$? Explain the reason for your decision.

3. A hockey club must make arrangements to book a hall for their wind-up. They have narrowed their choices to two different places. Hall A requires a rental fee of $600.00 plus $15.00 per person. Hall B requires $400.00 for a rental fee plus $20.00 per person. The following is a graph showing the cost of each hall.



x	Hall A	Hall B
30		
40		
50		
60		
70		
80		
90		

a) Write the equations for the total cost of each hall in which y is the total cost and x is the number of people who will attend the wind-up.

b) How many people would have to attend the wind-up for the total cost to be the same for both halls?

c) Create a table on your graphing calculator that begins with a minimum of 30 for the x-coordinate and then increases in increments of 10. Fill in the table (see above).

d) Under what circumstances would you choose each hall?

e) What would be the cost of each hall if 35 people were to attend? What would be the cost of each hall if 70 people were to attend? Describe two different ways to use your graphing calculator to answer these questions.

Exercise 2: Solving Problems (continued)

4. Jordan and Robert were looking for a job. They found two advertisements in their local newspaper. A clothing store was looking for a salesperson. It was offering a commission of 3% plus a monthly salary of $200.00. The other advertisement was for a sporting goods store also requiring a salesperson. This store was willing to pay a straight commission of 6% on all sales. Jordan got the job at the clothing store and Robert was hired at the sporting goods store.

   Clothing store's remuneration: $y = 0.03x + 200$

   Sporting goods store's remuneration: $y = 0.06x$

   a) Graph both equations on the same grid.

   b) How much would Jordan and Robert each need to sell monthly to get the same monthly salary?

   c) How much would they earn if they had $4500.00 worth of sales?

   d) How much would they earn if they had $8000.00 worth of sales?

5. Cab company A charges its customers a flat rate of $1.95 plus 90¢/km. Cab company B charges its customers a flat rate of $2.70 plus 75¢/km.

   a) Write an equation describing the cost of each company's fares.

   b) Use a graphing tool to graph each equation.

   c) What advice would you give someone who is deciding between the two companies?

6. Lindsay and Lucas live 455 km apart. Lindsay leaves her home at 12:00 noon, travelling 60 km/h towards Lucas's home. Simultaneously, Lucas starts from his home heading towards Lindsay and travelling 45 km/h. When will they meet?

7. In a certain steel mill, the cost in thousands of dollars of producing x tons of steel is given by the equation $C = 0.25x + 4$. The revenue in thousands of dollars from selling x tons is given by $R = 0.5x$.

   a) Graph each equation on the same grid.

   b) Find the break-even point.

   c) What value does the difference $R - C$ measure for all x greater than the x-coordinate of the break-even point?

8. The supply of a commodity (in thousands of units) is given by $S = 0.2p + 2$, where the dollars per unit is p. The demand for the commodity (in thousands of units) is given by the equation $D = -0.6p + 10$, where the dollars per unit is p.

   a) Graph each equation on the same grid.

   b) The value of p for which supply equals demand is called the equilibrium price. Determine the equilibrium price.

   c) What will happen at prices greater than this price?

9. A family is going to buy a new air-conditioning unit. One brand costs $1800 to buy, and $60/month to operate. A more expensive brand costs $2600 to buy but is more efficient, and costs only $50/month to operate.

   a) Write an equation to describe the total cost of the two air conditioning units.

   b) What advice would you give this family regarding this purchase?

Exercise 3: Spreadsheets

Use a spreadsheet to solve the following problems:

1. One number is 4 more than another number. Their sum is 62. Find the two numbers.
2. A piggy bank contains $270 in dimes and nickels. If there are 42 coins in all, how many dimes and how many nickels are there?

Exercise 4: Additional Word Problems

Solve the following problems using a graphing tool of your choice.

1. The weather office collects and analyzes data in order to predict weather. A hurricane on a path is given by $2x + 3y = 12$. At the same time, an oil tanker on the path is given by $4x - 3y = 6$.
   a) Plot the path of the hurricane and the tanker.
   b) Where will the two meet?
   c) If there is a port at $\left(2, \frac{2}{3}\right)$, can the ship make it to port before it enters the path of the hurricane?
2. Two bees have found a food source. The first bee's path back to the hive follows the line $y = \frac{4}{5}x$. The second bee's path follows the line $y = -2x + 3$. Their paths cross at the hive. Plot the paths of the bees to find the coordinates of the hive.
3. You are considering joining one of two fitness clubs. At Fitness Centre A there is no membership fee, but there is a user fee of $6.00 per visit. At Fitness Centre B, there is an annual fee of $75.00 and a user fee of $4.50 per visit. Explain how you would decide which club to join. Demonstrate your thinking using a mathematical equation.
4. One car sells for $22 000 and costs an average of $0.15 per km to maintain. Another car sells for $25,000 and costs an average of $0.10 per km to maintain. If each of these cars is driven the same number of kilometres, after how many kilometres would the total cost of the first car be the same as the total cost of the second car? Show your work.
5. A television addict can buy either two televisions and three VCRs for £1750 (British pounds) or four televisions and one VCR for £1250 (British pounds). Find the cost of each in
   a) pounds
   b) Canadian funds (refer to Attachment C-1: Foreign Cash Bulletin or check the daily rate at your local bank/credit union).
6. A forgery was suspected when a crown said to be gold was thought to contain some silver. The crown weighs 45 ounces and has a volume of 5 cubic inches. Silver weighs 6 ounces per cubic inch. Gold weighs 11 ounces per cubic inch. Is there silver mixed with the gold and, if so, how much?
7. Your math teacher tells you that next week's test is worth 141 points and contains 31 problems. Each problem is worth either 6 points or 3 points. Because you are studying systems of linear equations, your teacher says that for bonus marks you should be able to figure out how many problems of each value are on the test. How many of each value are there?

Exercise 4: Additional Word Problems (continued)

8. Antifreeze A is 18% alcohol. Antifreeze B is 10% alcohol. How many litres of each should be mixed to get 20 L of a mixture of 15% alcohol?

9. A limited edition coin produced by the mint was offered for sale. The cost was one coin for $12 or two coins for $20. The mint sold 880 coins and the total amount of money taken in was $9840. How many people ordered two coins?

10. Ed sells magazines part time. His combined sales for Thursday, Friday, and Saturday totalled $66. On Thursday, he sold $4 more than on Friday. On Saturday, he sold $7 more than on Thursday. How much did he take in each day?

11. Sheena picked raspberries on 3 days. She picked a total of 87 litres. On Tuesday, she picked 15 litres more than on Monday. On Wednesday, she picked 3 litres fewer than Tuesday. How many litres did she pick each day?

12. In a factory, there are three polishing machines, A, B, and C. When all three of them are working, 5800 lenses can be polished in one week. When only A and B are working, 3300 lenses can be polished in one week. When only B and C are working, 4100 lenses can be polished in one week. How many lenses can be polished in a week by each machine?

13. Tim, Deena, and Carlos can weld 37 linear feet per hour when working together. Tim and Deena together can weld 22 linear feet per hour, while Tim and Carlos together can weld 25 linear feet per hour. How many linear feet per hour can each weld alone?

14. Mr. Collinson invested $6000 into mutual funds. The first mutual fund had a 6% return and the other had a 5% return. How much was invested in each fund if Mr. Collinson's investments made $335?

15. A piggy bank contains $2.70 in dimes and nickels. There are 42 coins in all. How many dimes and how many nickels are there?

16. A tourist took a trip of 700 miles, driving at an average speed of 30 miles per hour before lunch and at an average speed of 50 miles per hour after lunch. His time on the road after lunch was 1 hour more than twice the time on the road before lunch. How many hours did he drive before stopping for lunch? How long was he on the road after lunch?

17. With a tail wind a plane travelled 120 km in 2 hr, but the return trip against the wind took 0.5 hr longer. Find the wind speed and the plane's air speed.

18. Four cartons of milk and three dozen eggs cost $4.85; five cartons of milk and two dozen eggs cost $4.40. Find the prices of a carton of milk and a dozen eggs.

19. A grocer has some coffee worth $1.69 per pound and some worth only $1.26 per pound. How many pounds of each kind of coffee should he mix together to get 100 pounds worth $1.53 per pound?

Attachment C-1

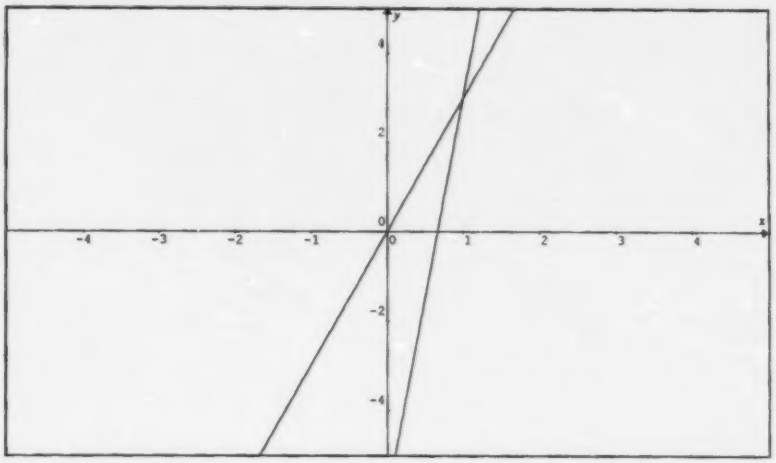
Foreign Cash Bulletin

Buying Suggested Counter Rate	Quotations Expressed in Canadian Dollars Foreign Currency	Selling Suggested Counter Rate
1.3424	UNITED STATES .....NOTES	1.3784
1.3319	UNITED STATES.....COINS	1.3889
2.0473	BANK OF ENGLAND .....NOTES	2.1573
2.0473	SCOTLAND.....POUND	2.1573
2.0473	IRELAND NORTH .....POUND	2.1573
2.0839	IRELAND EIRE PUNT .....POUND	2.2172
1.0309	AUSTRALIA .....DOLLAR	1.1557
0.2189	DENMARK .....KRONER	0.2421
0.4245	E. CARIBBEAN.....DOLLAR	0.5870
0.2652	FINLAND .....MARKKA	0.3627
0.2507	FRANCE.....FRANC	0.2744
0.8572	GERMANY .....D. MARK	0.9375
0.1620	HONG KONG .....DOLLAR	0.1856
0.7644	NETHERLANDS.....GUILDER	0.8283
0.1975	NORWAY .....KRONER	0.2184

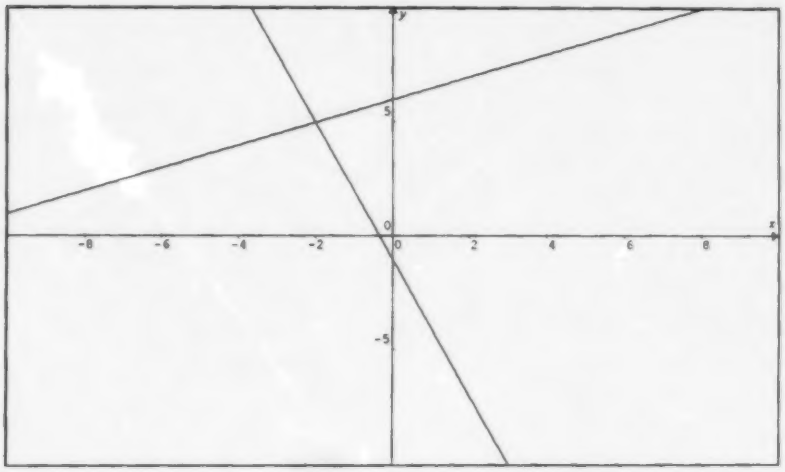
Unit C
Systems of Linear Equations
Solutions

Exercise 1: Solving Systems of Linear Equations — Solutions

1. a) $y = 9x - 6$
$y = 3x$
$(1, 3)$

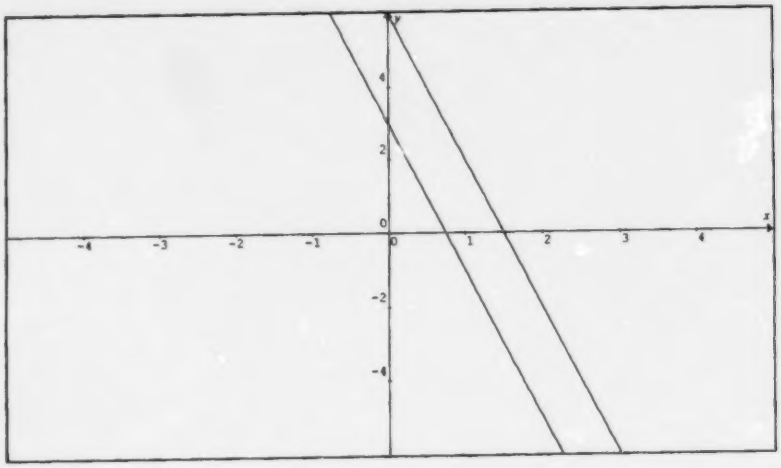


b) $2y = x + 12$
$y = -3x - 1$
$(-2, 5)$

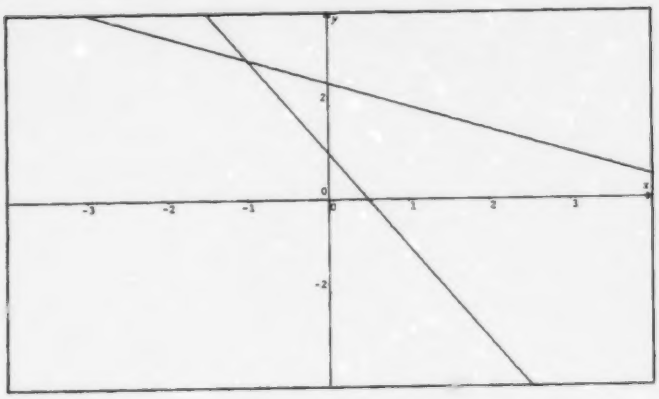


Exercise 1: Solving Systems of Linear Equations — Solutions (continued)

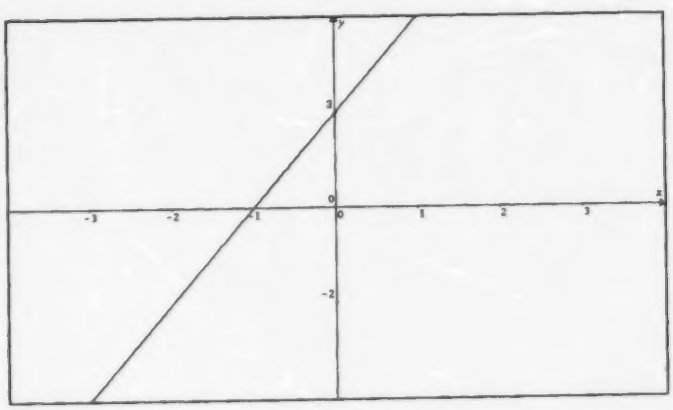
c) $y = -4x + 3$
$y = -4x + 6$
inconsistent system



d) $2x + y = 1$
$x + 2y = 5$
$(-1, 3)$

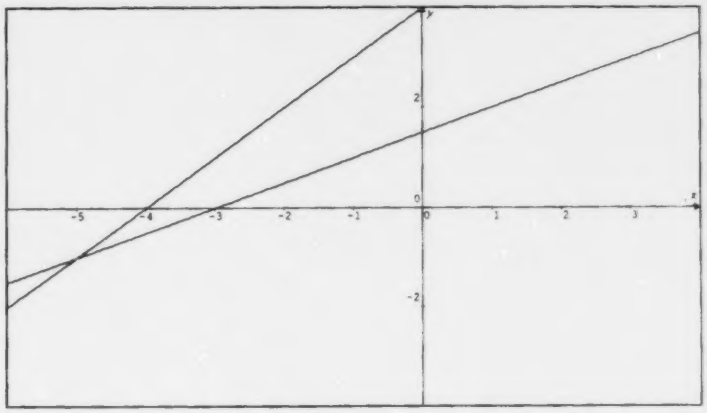


e) $y = 2(x + 3) - 4$
$y - 2x = 2$
dependent system



Exercise 1: Solving Systems of Linear Equations — Solutions (continued)

f) $2y = x + 3$
$y = x + 4$
$(-5, -1)$



2. Finding the y-intercept (TI-83)

Step 1: Press [y=] and enter the equation

Step 2: Press [2nd] (CALC) [0] [ENTER]

Finding the x-intercept

Step 1: Press [2nd] (CALC) [2]

Step 2: Move the cursor to the left of the x-intercept and press [ENTER]

Step 3: Move the cursor to the right of the x-intercept and press [ENTER] [ENTER]

a) x-intercept = 0
y-intercept = 0

b) x-intercept = 2
y-intercept = 4

c) x-intercept = −0.267
y-intercept = 4

d) x-intercept = −3
y-intercept = 1

e) x-intercept = 0.083
y-intercept = $-\frac{1}{4}$

f) x-intercept = 75
y-intercept = 30

g) x-intercept = 8
y-intercept = −4

h) x-intercept = 5
y-intercept = 15

Exercise 1: Solving Systems of Linear Equations — Solutions (continued)

3. Finding the point of intersection on a graphing calculator.

   Step 1: Press [Y=] and enter the first equation

   Step 2: Press [Y=] and enter the second equation

   Step 3: Press [2nd] (CALC) [5] [ENTER] [ENTER] [ENTER]

   a) (5, 5)
   b) (7.167, 15.667)
   c) (3.545, –2.818)
   d) no solution
   e) every point on the line
   f) (–8.667, –93.333)
   g) no solution
   h) every point on the line
   i) (–3, 8)

4. Answers will vary.

5. a) (5, 5)
   b) (5, 2)
   c) (3, 4)
   d) (–10, 2)
   e) (1.915, 1.319)
   f) (–6, 5)
   g) (–2, –3)
   h) every point on the line
   i) (–6, 5)
   j) no solution
   k) (36, –5)
   l) (–40, 20)

6. a) (–2, 3)
   b) (2, 2)
   c) (3, –1)
   d) (0.5, 0)
   e) (20, –8)
   f) (–1, –3)
   g) (1.571, 1.357)
   h) (–6, 3)
   i) (0, 0)

Exercise 2: Solving Problems — Solutions

1. Two possible answers.

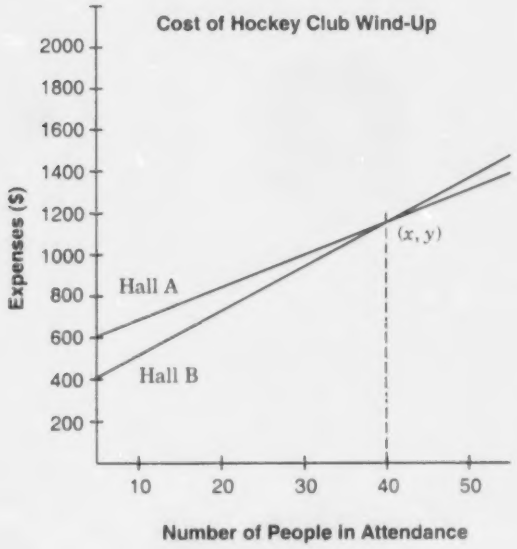
   $x + 2y = 4$ $\qquad$ $2x - y = 5.5$

   $x - 4y = 1$ $\qquad$ $x - 10y = -2$

2. The addition or subtraction method is the easiest method to use if you do not want to work with fractions.

3. a) $y = 15x + 600$
   $y = 20x + 400$

   b) 40 people



c)

x	Hall A	Hall B
30	1050	1000
40	1200	1200
50	1350	1400
60	1500	1600
70	1650	1800
80	1800	2000
90	1950	2200

d) If more than 40 people attend, then Hall A will be cheaper. Otherwise, it will be more economical to rent Hall B.

Exercise 2: Solving Problems — Solutions (continued)

e) If 35 people were to attend the function, it would cost \$1125 to rent Hall A and \$1100 to rent Hall B.

If 70 people were to attend the function, it would cost \$1650 to rent Hall A and \$1800 to rent Hall B.

A graphing calculator can be used to find these solutions. You can use the graph and find the point of intersection or you can go to the table function.

4. b) Monthly sales \$6666.67

   c) Jordan — \$335 and Robert — \$270

   d) Jordan — \$440 and Robert — \$480

5. a) $y = 0.90x + 1.90$
   $y = 0.75x + 2.10$

   c) If you are planning on going further than 5 km, company B is cheaper.

6. 4:20 p.m.

7. b) (16, 8000)

   c) R – C = Profit

8. b) (10, 4000)

   c) The demand will decrease.

9. b) $y = 60x + 1800$
   $y = 50x + 2600$

   c) After 80 months the more expensive unit will be cheaper.

Exercise 3: Spreadsheets — Solutions

1.

First Number	Second Number	Sum
25	29	54
26	30	56
27	31	58
28	32	60
29	33	62

2.

Dimes	Nickels	Total Worth
8	34	\$2.50
9	33	\$2.55
10	32	\$2.60
11	31	\$2.65
12	30	\$2.70

Exercise 4: Additional Word Problems — Solutions

1. $2x + 3y = 12$

$3y = -2x + 12$

$y = \frac{-2}{3}x + 4$

$4x - 3y = 6$

$-3y = -4x + 6$

$y = 4x + 6$

b) Point of intersection is (3, 2).

c) Yes, the tanker reaches port in time.

2. $-\frac{4}{5}x + y = 0$

$2x + y = 3$

$-4x + 5y = 0$

$4x + 2y = 6$

$7y = 6$

$y = \frac{6}{7}$

$2x + y = 3$

$2x + \frac{6}{7} = 3$

$2x = \frac{21}{7} - 67 = \frac{15}{7}$

$x = \frac{\frac{15}{7}}{2} = \frac{15}{14}$

The bees' home is at $\left(\frac{15}{14}, \frac{6}{7}\right)$

3. $6x = 4.5x + 75$

$1.5x = 75$

$x = 50$

You would have to visit 50 times to have an equal expenditure. If you did not go 50 times, Fitness Centre A is the better value. If you planned to go more often, then Fitness Centre B is the better value.

4. $0.15x + 22\,000 = 0.10 \times 25\,000$

$0.05x = 3000$

$x = 60\,000$

At 60 000 km, the total cost of the two cars is equal.

Exercise 4: Additional Word Problems — Solutions (continued)

5. $2t + 3v = 1750$
$4t + v = 1250$

$2(200) + 3v = 1750$
$3v = 1350$
$v = 450$ pounds

$2t + 3v = 1750$
$-12t - 3v = -3750$
$-10t = -2000$
$t = 200$ pounds

In Canadian funds: $200 \times 2.1573 = \$431.46$
$450 \times 2.1573 = \$970.79$

6. $g + s = 5$
$11g + 6s = 45$

$g + s = 5$
$3 + s = 5$
$s = 5 - 3 = 2$ cubic inches

$-6g - 6s = -30$
$11g + 6s = 45$
$5g = 15$
$g = \frac{13}{5}$ cubic inches

7. $x + y = 31$
$6x + 3y = 141$

$x + y = 31$
$16 + y = 31$
$y = 15$

$-3x - 3y = -93$
$6x + 3y = 141$
$3x = 48$
$x = 16$

There are 16 questions worth 6 points, and 15 questions worth 3 points.

8. $a + b = 20$
$18a + 10b = 15(20)$

$-10a + (-10b) = -200$
$18a + 10b = 300$
$8a = 100$
$a = 12.5$ L

$b = 20 - 12.5 = 7.5$ L

Exercise 4: Additional Word Problems — Solutions (continued)

9. $x + 2y = 880$ (x and y represent the number of people)
$12x + 20y = 9840$

$$\begin{aligned} -12x - 24y &= -10\,560 \\ 12x + 20y &= 9840 \\ \hline -4y &= -720 \end{aligned}$$

$y = 180$ people bought 2 coins

10. $t + f + s = 66$
$t - f = 4$
$-t + s = 7$

$f = 21 - 4 = \$17$
$s = 7 + 21 = \$28$

$$\begin{aligned} t + f + s &= 66 \\ t - f &= 4 \\ t - s &= -7 \\ \hline 3t &= 63 \end{aligned}$$

$t = \$21$

11. $m + t + w = 87$
$-m + t = 15$
$-t + w = -3$

$w = 35 - 3 = 32$ L
$m = 35 - 15 = 20$ L

$$\begin{aligned} m + t + w &= 87 \\ -m + t &= 15 \\ +t - w &= 3 \\ \hline 3t &= 105 \end{aligned}$$

$t = 35$ L

12. $a + b + c = 5800$
$a + b = 3300$
$b + c = 4100$

$a = 3300 - 1600 = 1700$ lenses
$c = 4100 - 1600 = 2500$ lenses

$$\begin{aligned} a + b + c &= 5800 \\ -a - b &= -3300 \\ -b - c &= -4100 \\ -b &= -1600 \end{aligned}$$

$b = 1600$ lenses

Exercise 4: Additional Word Problems — Solutions (continued)

13. t = 10 linear feet; d = 11 linear feet; c = 16 linear feet

14. $3500 invested at 6%; $2500 invested at 5%

15. 12 dimes; 30 nickels

16. 5 hours and 11 hours

17. Plane's speed: 54 km/h; Wind speed: 6 km/h

18. Milk: 50 cents; Eggs: 95 cents

19. 62.79 pounds ($1.69 coffee); 37.21 pounds ($1.26 coffee)

Unit D
Linear Programming

Teachers are advised to refer to *Senior 3 Applied Mathematics: A Foundation for Implementation* as their ***primary*** source of suggestions for instruction and assessment. The following exercises are an excellent resource if supplementary material is required.

Exercise 1: Introductory Problems

1. A manufacturer makes long-sleeve and short-sleeve t-shirts. Each long sleeve shirt requires 4 minutes on the cutting machine and 3 minutes on the stitching machine. Each short sleeve shirt requires 3 minutes on the cutting machine and one minute on the stitching machine. The cutting machine is available only 2 hours each day, and the stitching machine is available only 1 hour each day. If the profit on a short sleeve is $0.60 and $1.10 on a long sleeve, how many of each should be produced each day?

   Open up the spreadsheet file SHIRTS provided by your teacher. You should see the spreadsheet below:

	A	B	C	D	E
1		number	cut time	stitch time	profit
2	long-sleeve shirts	10	40	30	$11.00
3	short-sleeve shirts	20	60	20	$12.00
4	total		100	50	$23.00

   As you can see, in this example, 10 long-sleeve shirts and 20 short sleeve shirts produce $23.00 of profit. However, the machines have not had all their time used up. Change only the values in B2 and B3 to find out how many long-sleeve shirts and short-sleeve shirts should be produced. Remember to keep the machine times under the limits.

   long-sleeve shirts _____ short-sleeve shirts _____

   In your notebook, copy this spreadsheet and fill in the formulas in each cell for reference in the next problem.

	A	B	C	D	E
1		number	cut time	stitch time	profit
2	long-sleeve shirts				
3	short-sleeve shirts				
4	total				

Exercise 1: Introductory Problems (continued)

2. For this problem, you should set up your own spreadsheet that looks like this:

	A	B	C	D	E
1		number	electronic manufacturing	assembly	profit
2	standard				
3	deluxe				
4	total				

The Alford Computer Company manufactures two types of computers — standard and deluxe. In a day, 30 hours of electronic manufacturing time and 70 hours of assembly time are available. The standard model requires 1.5 hours of electronic manufacturing time and one hour of assembly time. The deluxe model requires 30 minutes of electronic manufacturing time and 2 hours of assembly time. The profit on the standard is $250.00 and $180.00 on the deluxe. How many of each model should be produced to achieve a maximum profit? What is the maximum profit?

Use the formulas from problem 2 to assist you in setting up your spreadsheet.

Exercise 2: Linear Inequalities

Solve and graph on a number line.

1. $-4x \leq 12$
2. $-8x > -32$
3. $40 < 8x$
4. $4 > x - 3$
5. $5 \leq x + 7$
6. $-4 \geq x - 6$
7. $6 - 6x < 30$
8. $6x - 6 > 9x$
9. $5x + 27 \leq 2x$
10. $9x - 2 > 7x + 12$
11. $22 - 3x > 30$
12. $\frac{2}{3}x < 18$
13. $8 - 2(2x + 1) < 2x - 3$
14. $8x - \frac{3}{4}(12x + 28) \leq 0$
15. $\frac{3(5x + 6)}{-4} < \frac{8 - 5x}{2}$
16. Banks, credit unions, and trust companies have certain guidelines they follow when lending money for mortgages. One of these indicates how large a mortgage payment plus monthly property tax a person can afford based on his or her monthly income. This is:

$$\text{Mortgage + Taxes} = \frac{1}{3}\ \text{Monthly Income}$$

a) If Kamara's monthly mortgage payment is $1322.00 and the monthly property taxes are $108.00, then what is the monthly minimum income she must have?

b) Robert has a job that pays $2200/month net. His wife earns $2350/month net. If the property taxes for a house they wish to purchase are $172/month, then what is the largest mortgage payment for which they qualify?

Exercise 3: Graphing Inequalities in a Plane

Using a graphing tool, graph the following:

1. $2x - 3y \leq 6$
2. $3x + 3y < 9$
3. $5x - 2y \geq 16$
4. $6x - 4y \geq 24$
5. $2x - 7y > 35$
6. $-5x - 2y \leq 14$
7. $x \geq 7$
8. $y > 4$
9. $y \leq -3$
10. $x \leq 3$
11. $\frac{1}{3}x + \frac{1}{2}y \leq -4$
12. $\frac{x + 2y}{3} \geq -2$

Exercise 4: Graphing and Solving Systems of Linear Inequalities

1. Using a graphing tool, graph the following systems of inequalities.

a) $y \leq 2x + 3$
$y > 4x - 1$

b) $2x + 3y \geq 9$
$-3x + 4y \geq -4$

c) $2x + y > 3$
$y \geq x$

d) $y \leq x - 3$
$-2y \leq 2x + 4$

e) $-x - y \leq 2$
$y - 2x > 1$

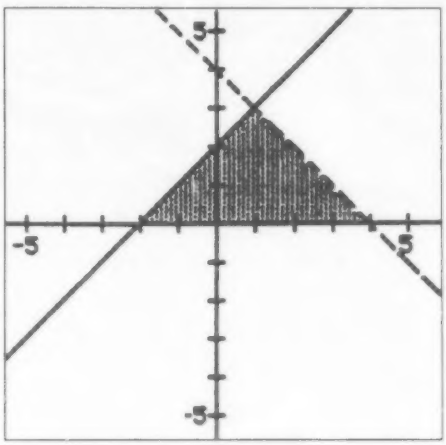
f) $3x - 4y < -12$
$3x + 4y > 0$

g) $y \leq -2x + 4$
$x > -3$
$y \geq 1$

h) $-x + 3y < 5$
$3x - 2y \geq 6$
$-4x + y < 3$

2. For each region, write the defining system of inequalities.

a)



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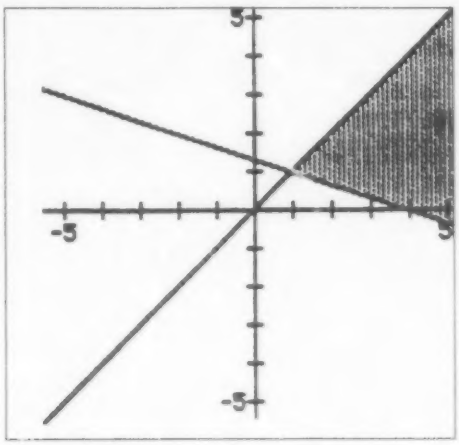

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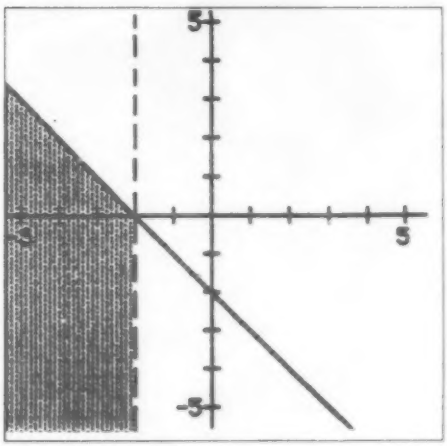
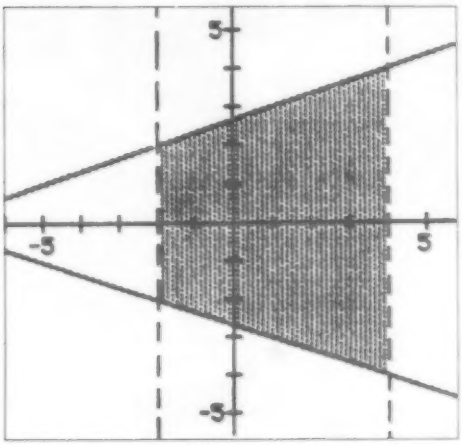
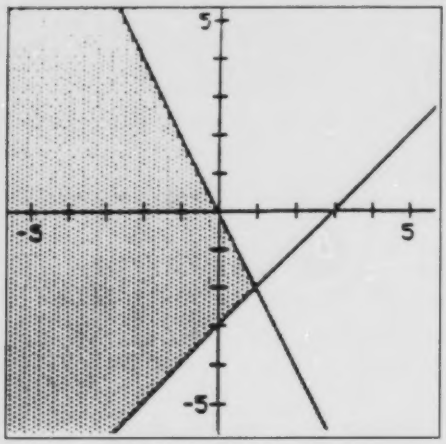


d)



Exercise 4: Graphing and Solving Systems of Linear Inequalities (continued)

e)



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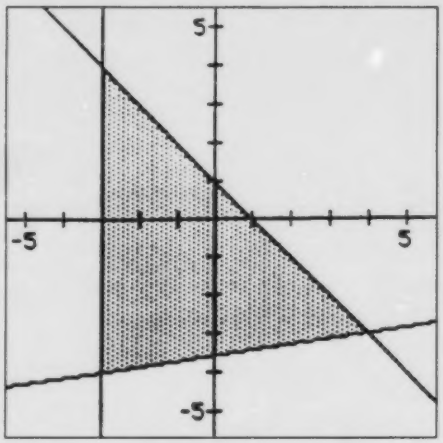
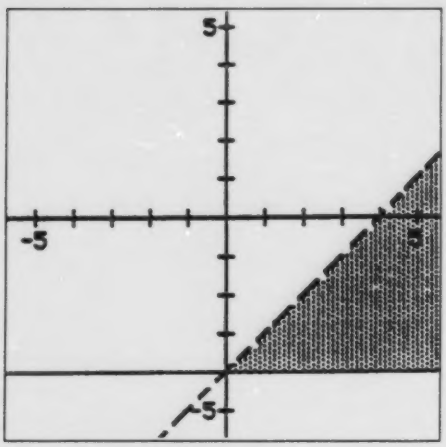
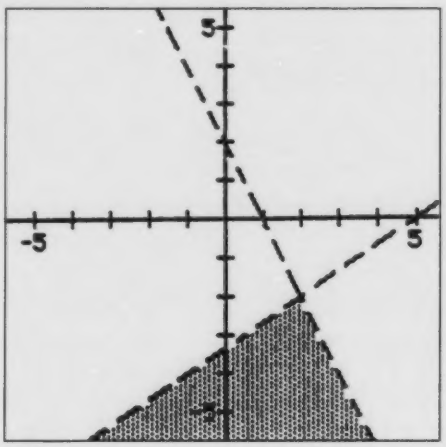


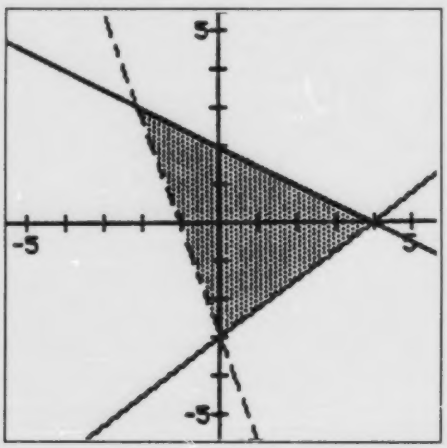
g)



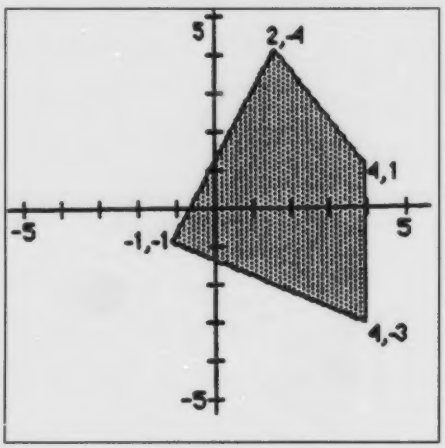
h)



i)



j)



Exercise 5: Solving Linear Programming Problems

1. A pet-store farmer has turtles and rabbits. He has room for only 100 pets. He can sell turtles for $10 each and rabbits for $30 each. He decides that he must earn at least $1500 to make the job worthwhile.
   a) Write the inequality that represents the total number of pets.
   b) Write the inequality that represents the total profit.
   c) Graph these inequalities.
   d) What are some possible solutions for the farmer?

2. A rectangle to be constructed has a length less than twice its width, its perimeter is at most 48 units, and its area is at least 32 square units.
   a) Write the inequality that represents the relationship between the length and width.
   b) Write the inequality that represents the perimeter.
   c) Write the inequality that represents the area.
   d) Graph these inequalities.
   e) Give two possible solutions for the dimensions of the rectangle.

3. A manufacturer makes long-sleeve and short-sleeve t-shirts. Each long-sleeve shirt requires 4 minutes on the cutting machine and 3 minutes on the stitching machine. Each short-sleeve shirt requires 3 minutes on the cutting machine and 1 minute on the stitching machine. The cutting machine is available only 2 hours each day, and the stitching machine is available only 1 hour each day. If the profit on a short-sleeve shirt is $0.60 and $1.10 on a long-sleeve shirt, how many of each should be produced each day?
   a) Write the inequality for the cutting machine.
   b) Write the inequality for the stitching machine.
   c) Graph the inequalities.
   d) What are two possible solutions?
   e) The best solution is 12 long sleeve and 24 short sleeve. What do you notice about the location of the best solution on the graph?

4. The Alford Computer Company manufactures two types of computers — standard and deluxe. In a day, there are 30 hours of electronic manufacturing time and 70 hours of assembly time available. The standard model requires 1.5 hours of electronic manufacturing time and 1 hour of assembly time. The deluxe model requires 30 minutes of electronic manufacturing time and 2 hours of assembly time. The profit on the standard is $250.00 and $180.00 on the deluxe. How many of each model should be produced to achieve a maximum profit? What is the maximum profit?
   a) Write the inequality for the electronic manufacturing time.
   b) Write the inequality for the assembly time.
   c) Graph the inequalities.
   d) What are two possible solutions?
   e) The best solution is 10 standard and 30 deluxe. What do you notice about the location of the best solution?

Exercise 5: Solving Linear Programming Problems (continued)

5. A zoo is given a sum of money to spend on monkeys and rabbits. The zoo must spend at least $2000 on the purchase to qualify for the grant. At least 14 animals must be bought. Each rabbit costs $40.00 and each monkey costs $400.00.
   a) Write the inequalities.
   b) Graph the inequalities.
   c) What is the best way the zoo can spend the money?
   d) Suppose that it costs $1.00/day to feed a rabbit and $5.00/day to feed a monkey. Write the cost equation. How much will it cost to feed all the rabbits and monkeys they purchase?
6. A construction equipment company makes a light duty (4-wheel) van that retails for $30 000 and a heavy duty (6 wheel) van that retails for $40 000. Each day the plant receives 300 wheels and 60 engines that can be installed in either type of van. Find the number of units that should be produced to give the greatest amount of sales. What is the greatest amount of sales?
7. A neighbourhood convenience store keeps two brands of potato chips in stock. The shelving can hold 240 bags total. Munchies will sell 3 times as much as Crispios. However, the profit on Munchies is $1.50 and the profit on Crispios is $2.00. How many bags of each should be stocked for maximum profit?
8. A small manufacturing company makes chairs and foot stools. Each chair requires 7 feet of lumber, each foot stool 3 feet. A chair requires 8 hours to make, while a foot stool takes 2 hours. The profit on a chair is $30.00 and $10.00 on a foot stool. If 420 feet of lumber and 400 hours of labour are available, how many of each should be produced to obtain a maximum profit?
9. Lawn fertilizer is produced in two types — regular and deluxe. Regular contains 3 lbs of nitrogen, 4 lbs of phosphoric acid, and 1 lb of potash. The deluxe bag contains 2 lbs of nitrogen, 6 lbs of phosphoric acid, and 3 lbs of potash. A knowledgeable homeowner has her lawn tested and it is determined that it requires at least 10 lbs of nitrogen, 20 lbs of phosphoric acid, and 7 lbs of potash. A bag of regular fertilizer is $3.00, and the deluxe is $4.00. How many bags of each should she buy to mix together, keeping her cost to a minimum?
10. Marvin's Greenhouses has a 100 m^2 plot of land available for vegetable plants. Marvin decides to plant peppers and carrots. He has budgeted $4000 for this particular section. The peppers will cost $\$30/m^2$ to grow and will generate $\$35/m^2$ in gross income. The carrots cost $\$50/m^2$ to grow and generate $\$65/m^2$ in gross income. How much of each should he plant?
11. Two men, John and Bill, make math supplies for schools. In 1 hour, John can make 10 protractors, 5 compasses, and 5 metre sticks. Bill can make 5 protractors, 5 compasses, and 20 metre sticks in the same time. They have an order for 40 protractors, 80 compasses, and 110 metre sticks. John earns $8.50 per hour, and Bill earns $12.00 per hour. How long should each work to minimize the cost of labour?
12. Ivan and his daughter service lawn equipment such as lawn mowers and weed trimmers. Ivan can service a lawn mower in 15 minutes and a weed trimmer in 12 minutes. His daughter can service a lawn mower in 10 minutes and a weed trimmer in 20 minutes. Ivan is paid $12.00/hour and his daughter is paid $8.00/hour. If they have an order for 50 lawn mowers and 40 weed trimmers, how long should each work to minimize the cost of labour?

Exercise 5: Solving Linear Programming Problems (continued)

13. A manufacturer makes two products. They are manufactured according to the following table.

	Moulding Time	Painting Time	Finishing Time
Product A	3 hours	4 hours	1 hour
Product B	3 hours	2 hours	2 hours

Each week 210 hours are available in moulding, 200 hours in painting, and 120 hours in finishing. The shipping department can only handle 40 units of product A per week, but an unlimited number of product B. The profit is as follows:

Product A: $20.00

Product B: $30.00

Write a presentation to the plant supervisor detailing how you recommend the production be set up. Include a covering memo, graphs, and the logic behind your suggestion. You may use a computer to generate your work.

Unit D
Linear Programming
Solutions

Teaching notes are provided with this unit (see Attachment D-1). Note that Exercise 5: Solving Linear Programming Problems may be used for test or exam questions.

Exercise 1: Introductory Problems — Solutions

These problems are designed to be done by students on a spreadsheet. The problems allow students to have some idea of where the unit is heading. The teacher is to provide the students with a spreadsheet similar to the one below called SHIRTS. They are to change the number of shirts only.

	A	B	C	D	E
1		number	cut time	stitch time	profit
2	long sleeve	10	=4*B2	=3*B2	=1.1*B2
3	short sleeve	20	=3*B3	=B3	=0.6*B3
4	total	=B2+B3	=C2+C3	=D2+D3	=E2+E3

The students should copy down the formulas used in the SHIRTS spreadsheet to use as guides when they create their own spreadsheet in the second introductory problem.

1.

Cell	Formula
C2	=4*B2
C3	=3*B3
C4	=C2+C3
D2	=3*B2
D3	=1*B3
D4	=D2+D3
E2	=1.1*B2
E3	=0.6*B3
E4	=E2+E3

Answer: 12 long, 24 short = $27.60 profit

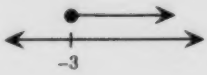
2.

Cell	Formula
C2	=1.5*B2
C3	=0.5*B3
C4	=C2+C3
D2	=1*B2
D3	=2*B3
D4	=D2+D3
E2	=250*B2
E3	=180*B3
E4	=E2+E3

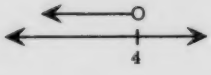
Answer: 10 standard, 30 deluxe = $7900 profit

Exercise 2: Linear Inequalities — Solutions

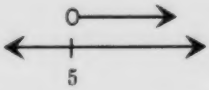
1. $x \geq -3$



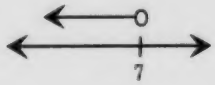
2. $x < 4$



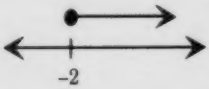
3. $x > 5$



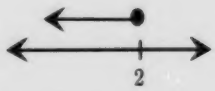
4. $x < 7$



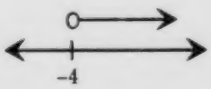
5. $x \geq -2$



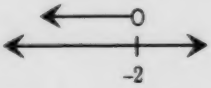
6. $x \leq 2$



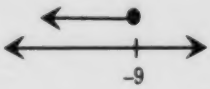
7. $x > -4$



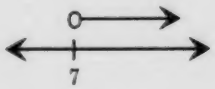
8. $x < -2$



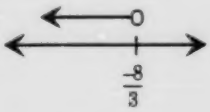
9. $x \leq -9$



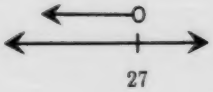
10. $x > 7$



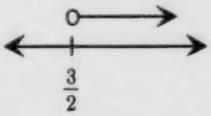
11. $x < -\frac{8}{3}$



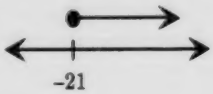
12. $x < 27$



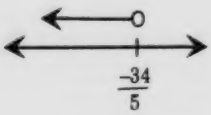
13. $x > \frac{3}{2}$



14. $x > -21$

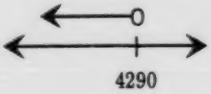


15. $x < \frac{-34}{5}$ or -6.8



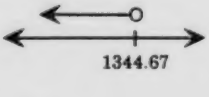
16. a) $1322 + 108 < \frac{1}{3}x$

$x > 4290$



b) $172 + x < \frac{1}{3}(2200 + 2350)$

$x < 1344.67$



Exercise 3: Graphing Inequalities in a Plane — Solutions

This section is included to reinforce the $y = mx + b$ form and algebraic manipulation.

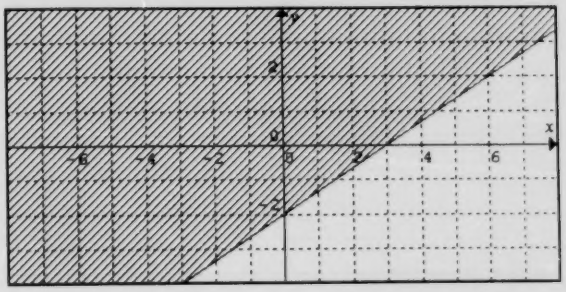
1. $2x - 3y \le 6 \rightarrow \quad 2x - 3y = 6$

$$y \ge \frac{2}{3}x - 2$$

test (0, 0)

$2(0) - 3(0) \le 6$

$0 \le 6$ yes



2. $3x + 3y < 9 \rightarrow \quad 3x + 3y = 9$

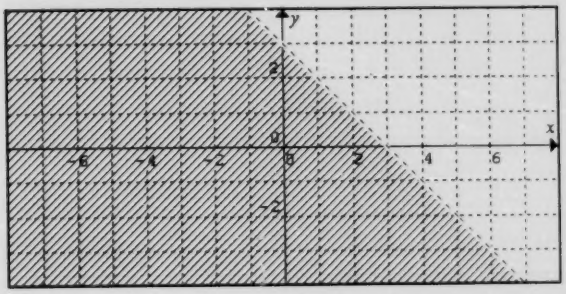
$$y = -x + 3$$

$$y < -x + 3$$

test (0, 0)

$3(0) + 3(0) < 9$

$0 < 9$ yes



3. $5x - 2y \ge 16 \rightarrow \quad 5x - 2y = 16$

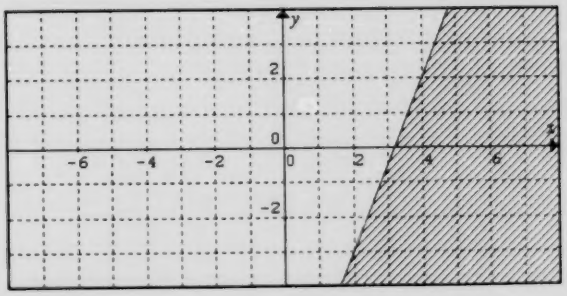
$$y = \frac{5}{2}x - 8$$

$$y \le \frac{5}{2}x - 8$$

test (0, 0)

$5(0) - 2(0) \ge 16$

$0 \ge 16$ no



4. $6x - 4y \ge 24 \rightarrow \quad 6x - 4y = 24$

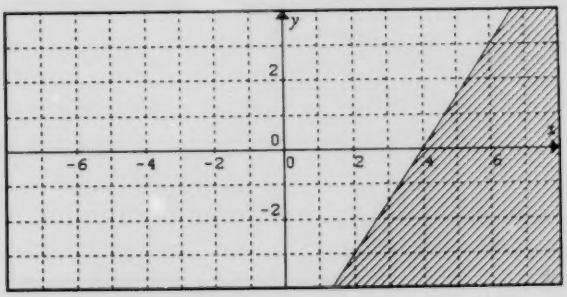
$$y = \frac{6}{4}x - 6$$

$$y \le \frac{3}{2}x - 6$$

test (0, 0)

$6(0) - 4(0) \ge 24$

$0 \ge 24$ no



Exercise 3: Graphing Inequalities in a Plane — Solutions (continued)

5. $2x - 7y > 35 \rightarrow 2x - 7y = 35$

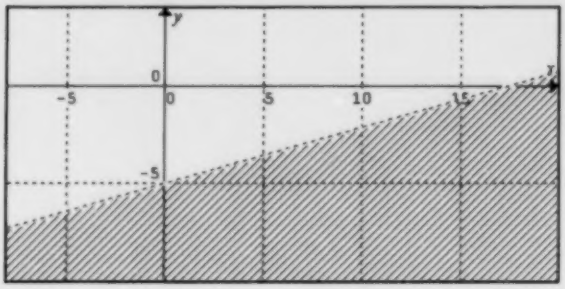
$$y = \frac{2}{7}x - 5$$

$$y < \frac{2}{7}x - 5$$

test (0, 0)

$2(0) - 7(0) > 35$

$0 > 35$ no



6. $-5x - 2y \leq 14 \rightarrow -5x - 2y = 14$

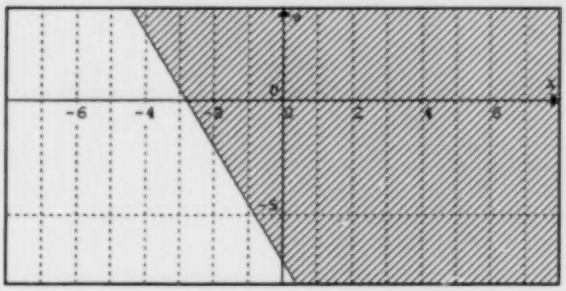
$$y = \frac{-5}{2}x - 7$$

$$y \geq \frac{-5}{2}x - 7$$

test (0, 0)

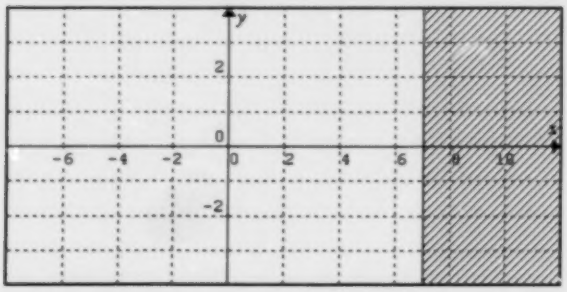
$-5(0) - 2(0) \leq 14$

$0 \leq 14$ yes



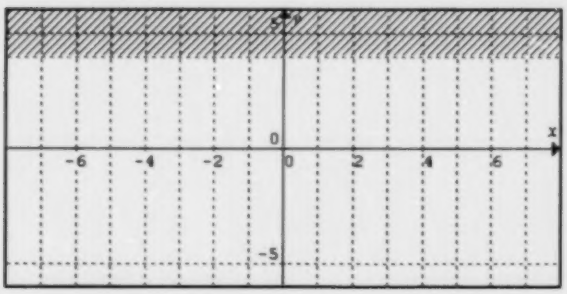
7. $x \geq 7 \rightarrow x = 7$

$x \geq 7$



8. $y > 4 \rightarrow y = 4$

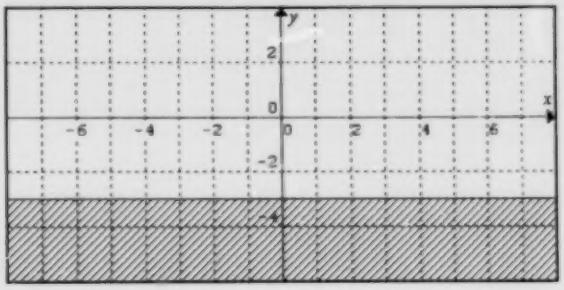
$y > 4$



Exercise 3: Graphing Inequalities in a Plane — Solutions (continued)

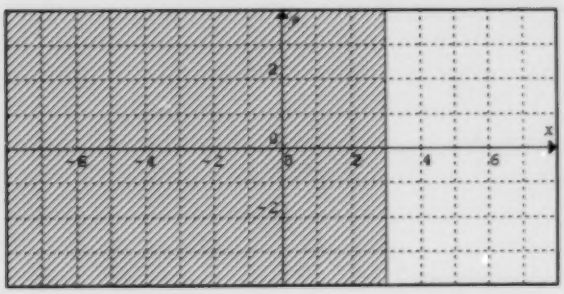
9. $y \le -3 \rightarrow y = -3$

$y \le -3$



10. $x \le 3 \rightarrow x = 3$

$x \le 3$



11. $\frac{1}{3}x + \frac{1}{2}y \le -4 \rightarrow \frac{1}{3}x + \frac{1}{2}y = -4$

$2x + 3y = -24$

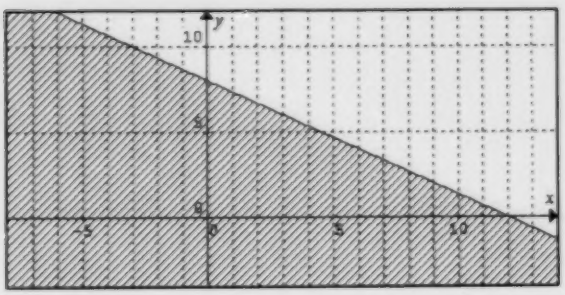
$y = \frac{-2}{3}x - 8$

$y \le \frac{-2}{3}x - 8$

test (0, 0)

$\frac{1}{3}(0) + \frac{1}{2}(0) \le -4$

$0 \le -4$ no



12. $\frac{x + 2y}{3} \ge -2 \rightarrow \frac{x + 2y}{3} = -2$

$x + 2y = -6$

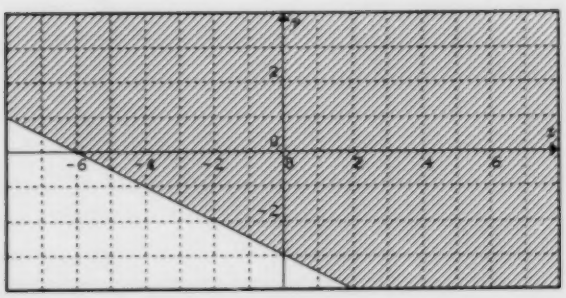
$y = -\frac{1}{2}x - 3$

$y \ge -\frac{1}{2}x - 3$

test (0, 0)

$\frac{0 + 2(0)}{3} \ge -2$

$0 \ge -2$ yes



Exercise 4: Graphing and Solving Systems of Linear Equations — Solutions

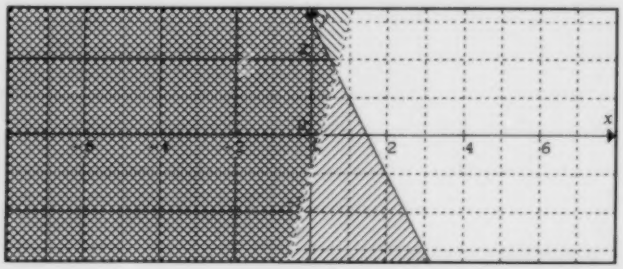
1. a) (1) $y \le -2x + 3 \rightarrow y = -2x + 3$ (2) $y > 4x - 1 \rightarrow y = 4x - 1$

test $(0, 0)$ test $(0, 0)$

$0 \le -2(0) + 3$ $0 > 4(0) - 1$

$0 \le 3$ yes $0 > -1$ yes

$y \le -2x + 3,\ y > 4x - 1$



b) (1) $2x + 3y \ge 9 \rightarrow 2x + 3y = 9$ (2) $-3x + 4y \ge -4 \rightarrow -3x + 4y = -4$

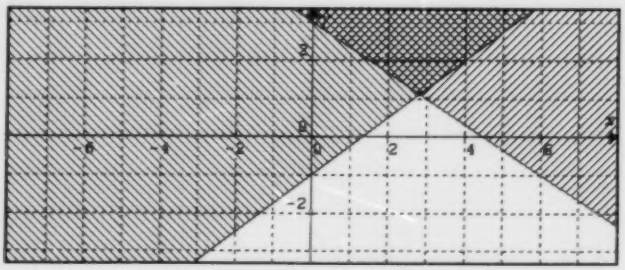
$y = \frac{-2}{3}x + 3$ $y = \frac{3}{4}x - 1$

test $(0, 0)$ test $(0, 0)$

$2(0) + 3(0) \ge 0$ $-3(0) + 4(0) \ge -4$

$0 \ge 9$ no $0 \ge -4$ yes

$y \ge \frac{-2}{3}x + 3, y \ge \frac{3}{4}x - 1$



Exercise 4: Graphing and Solving Systems of Linear Equations — Solutions

c) (1) $2x + y > 3 \rightarrow 2x + y = 3$

$y = -2x + 3$

test $(0, 0)$

$2(0) + 0 > 3$

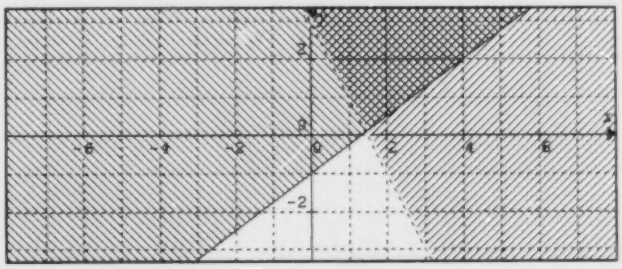
$0 > 3$ no

(2) $y \geq x \rightarrow y = x$

test $(-1, 1)$

$1 \geq -1$ yes

$y > -2x + 3,\ y \geq x$



d) (1) $y \leq x - 3 \rightarrow y = x - 3$

test $(0, 0)$

$0 \leq 0 - 3$

$0 \leq -3$ no

(2) $-2y \leq 2x + 4 \rightarrow -2y = 2x + 4$

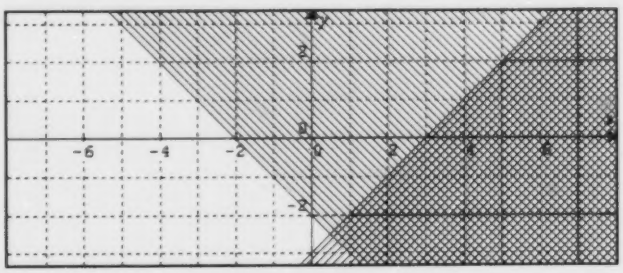
$y = -x - 2$

test $(0, 0)$

$-2(0) \leq 2(0) + 4$

$0 \leq 4$ yes

$y \leq x - 3,\ y \geq -x - 2$



Exercise 4: Graphing and Solving Systems of Linear Equations — Solutions

e) (1) $-x - y \leq 2 \rightarrow -x - y = 2$ (2) $y - 2x > 1 \rightarrow y - 2x = 1$

$y = -x - 2$ $y = 2x + 1$

test $(0, 0)$ test $(0, 0)$

$-0 - 0 \leq 2$ $0 - 2(0) > 1$

$0 \leq 2$ yes $0 > 1$ no

$y \geq -x - 2,\ y > 2x + 1$

f) (1) $3x - 4y < -12 \rightarrow 3x - 4y = -12$ (2) $3x + 4y > 0 \rightarrow 3x + 4y = 0$

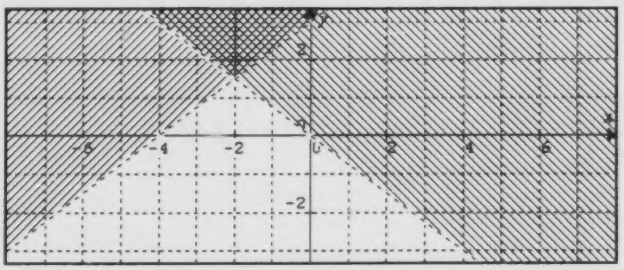
$y = \frac{3}{4}x + 3$ $y = -\frac{3}{4}x$

test $(0, 0)$ test $(1, 1)$

$3(0) - 4(0) < -12$ $3(1) + 4(1) > 0$

$0 < -12$ no $7 > 0$ yes

$y > \frac{3}{4}x + 3,\ y > -\frac{3}{4}x$



Exercise 4: Graphing and Solving Systems of Linear Equations — Solutions

g) (1) $y \le -2x + 4 \rightarrow y = -2x + 4$ (2) $x > -3 \rightarrow x = -3$

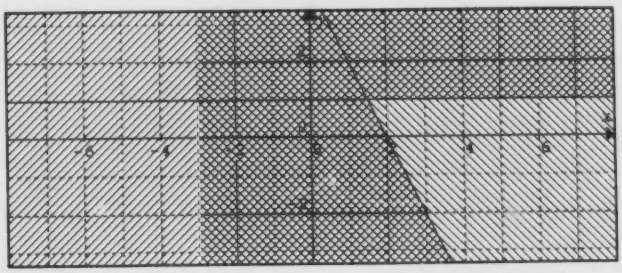
(4) $y \ge 1 \rightarrow y = 1$

test $(0, 0)$

$0 \le -2(0) + 4$

$0 \le 4$ yes

$y \le -2x + 4, x > -3, y \ge 1$



h) (1) $-x + 3y < 5 \rightarrow -x + 3y = 5$ (2) $3x - 2y \ge 6 \rightarrow 3x - 2y = 6$

$y = \frac{1}{3}x + \frac{5}{3}$ $y = \frac{3}{2}x - 3$

test $(0, 0)$ test $(0, 0)$

$-0 + 3(0) < 5$ $3(0) - 2(0) \ge 6$

$0 < 5$ yes $0 \ge 6$ no

(3) $-4x + y < 3 \rightarrow -4x + y = 3$

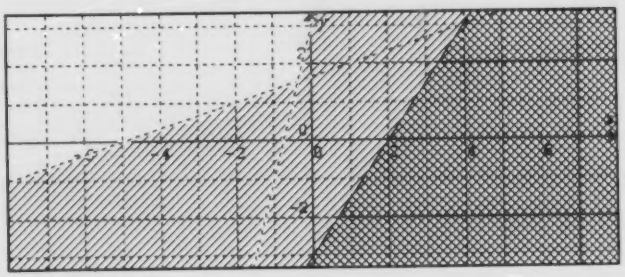
$y = 4x = 3$

test $(0, 0)$

$-4(0) + 0 < 3$

$0 < 3$ yes

$y < \frac{x}{3} + \frac{5}{3}, y \le \frac{3}{2}x - 3, y < 4x + 3$



Exercise 4: Graphing and Solving Systems of Linear Equations — Solutions

2. a) $y \leq x + 2$
$y < -x + 4$
$y \geq 0$

b) $y \leq x$
$y \geq -\frac{1}{3}x + \frac{4}{3}$

c) $y \leq -x - 2$
$x < -2$

d) $x > -2$
$x < 4$
$y \leq \frac{1}{3}x + \frac{8}{3}$
$y \geq -\frac{1}{3}x - \frac{8}{3}$

e) $y \geq x - 3$
$y \leq -2x$

f) $x \geq -3$
$y \leq -x + 1$
$y \geq \frac{1}{7}x - \frac{25}{7}$

g) $y \geq -4$
$y < x - 4$

h) $y < -2x + 2$
$y < \frac{2}{3}x - \frac{10}{3}$

i) $y \leq -\frac{1}{2}x + 2$
$y > -3x - 3$
$y \geq \frac{3}{4}x - 3$

j) $y \leq \frac{5}{3}x + \frac{2}{3}$
$y \leq -\frac{3}{2}x + 7$
$y \leq 4$
$y \geq -\frac{2}{5}x - \frac{7}{5}$

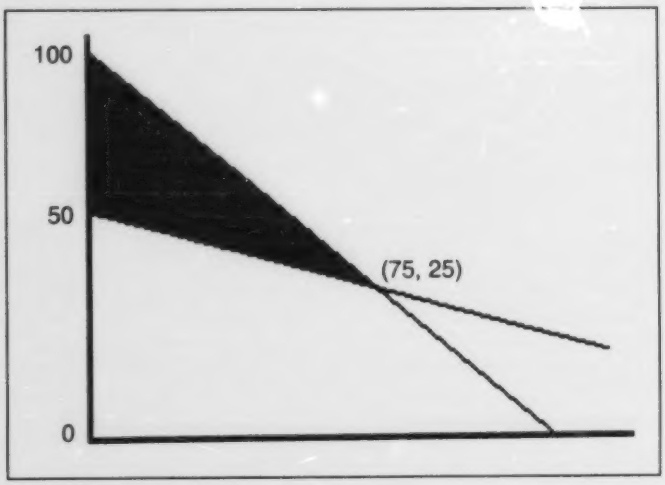
Exercise 5: Solving Linear Programming Problems — Solutions

Teaching Note: The last part in each problem allows the student to discover that the best solution occurs at a vertex of the feasibility area.

1. a) $x + y \leq 100$

   b) $10x + 30y \geq 1500$

   c)



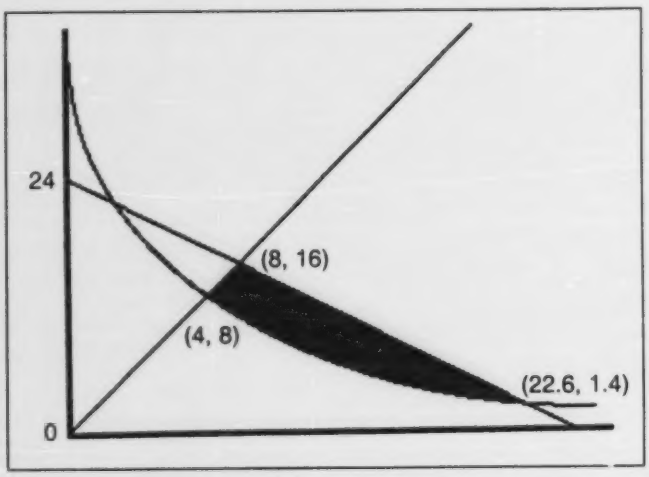
   d) Solutions include any points in the shaded area.

2. a) $y < 2x$

   b) $2x + 2y \leq 48$

   c) $xy \geq 32$

   d)



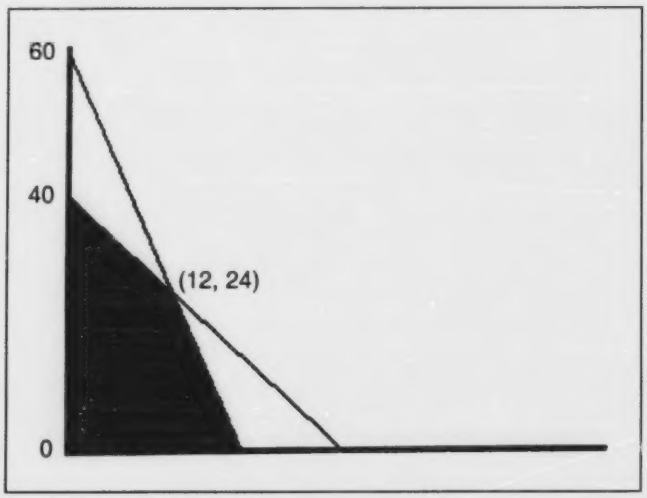
   e) Solutions include any points in the shaded area.

Exercise 5: Solving Linear Programming Problems — Solutions (continued)

3. a) $4x + 3y \leq 120$

   b) $3x + y \leq 60$

   c)



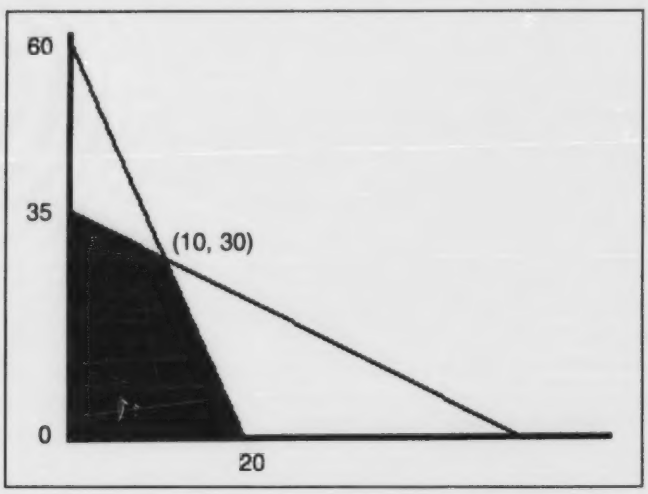
   d) Solutions include any points in the shaded area.

   e) The best solution is the point where the two lines intersect.

4. a) $1.5x + 0.5y \leq 30$

   b) $x + 2y \leq 70$

   c)



   d) Solutions include any points in the shaded area.

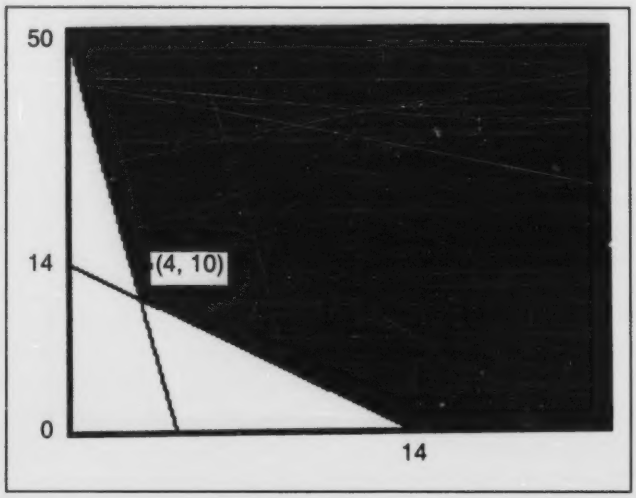
   e) The best solution is the point where the two lines intersect.

Exercise 5: Solving Linear Programming Problems — Solutions (continued)

5. a) $x + y \geq 14$

   $400x + 40y \geq 2000$

   b)

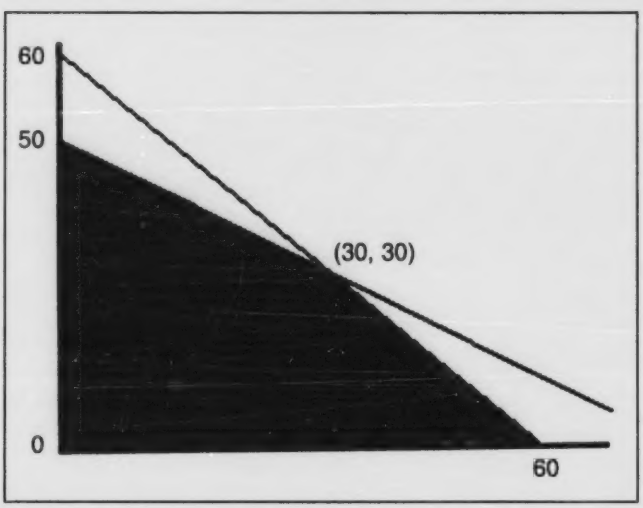


   c) Cost = $1x + 5y$

   The best solution is 4 monkeys and 10 rabbits. This combination costs $30.00/day to feed.

6. $4x + 6y \leq 300$

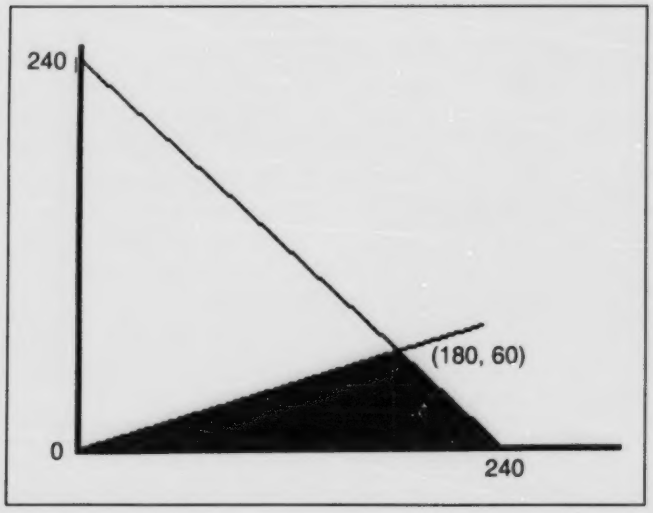
   $x + y \leq 60$



The best solution is 30 light duty and 30 heavy duty for sales of $2.1 million.

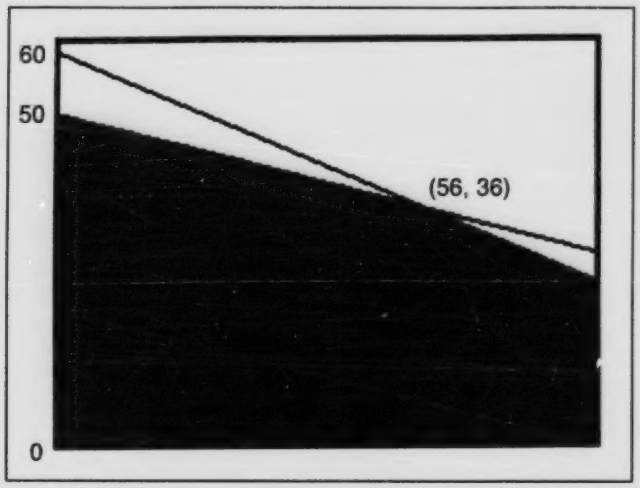
Exercise 5: Solving Linear Programming Problems — Solutions (continued)

7. $x + y \leq 240$
   $x \geq 3y$



The best solution is 180 of Munchies and 60 of Crispios. Profit is $390.00.

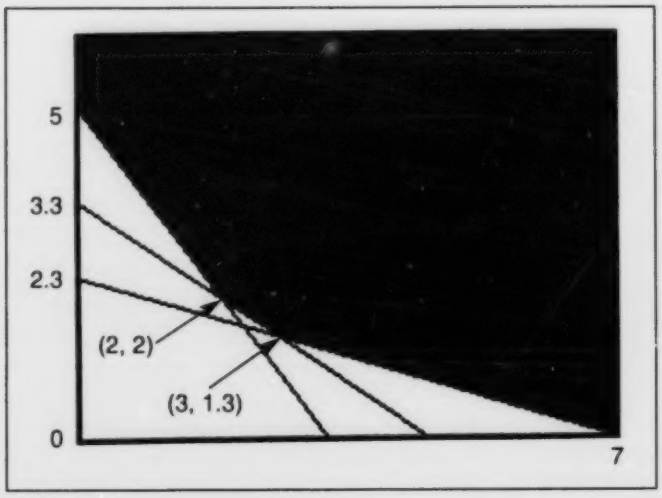
8. $3x + 7y \leq 420$
   $2x + 8y \leq 400$



The best solution is 56 foot stools and 36 chairs. Maximum profit is $1640.

Exercise 5: Solving Linear Programming Problems — Solutions (continued)

9. $3x + 2y \geq 10$
   $4x + 6y \geq 20$
   $x + 3y \geq 7$

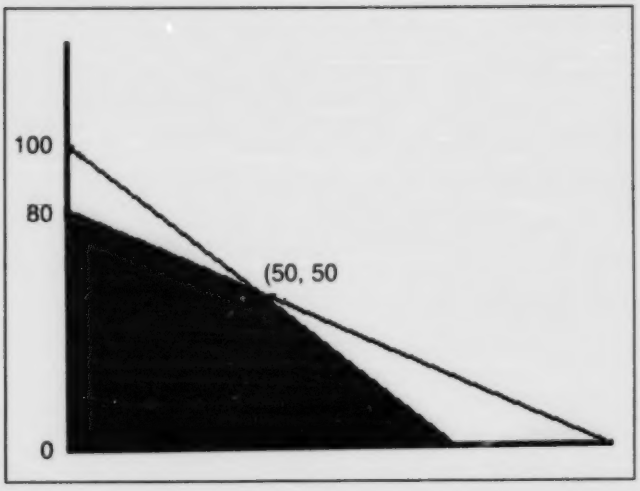


Possible points of intersection are (2, 2) and (3, ¾).

Two bags of each would cost \$14.00.

Three bags of regular and two deluxe (cannot buy $\frac{4}{3}$ bags) would cost \$17.00.

10. $x + y \leq 100$
    $30x + 50y \leq 4000$



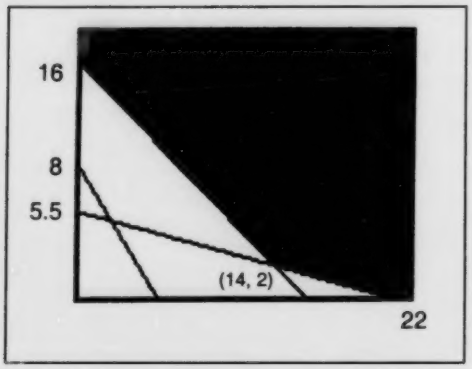
50 m^2 — peppers
50 m^2 — carrots
Profit is \$1000.

Exercise 5: Solving Linear Programming Problems — Solutions (continued)

11. $10x + 5y \geq 40$

    $5x + 5y \geq 80$

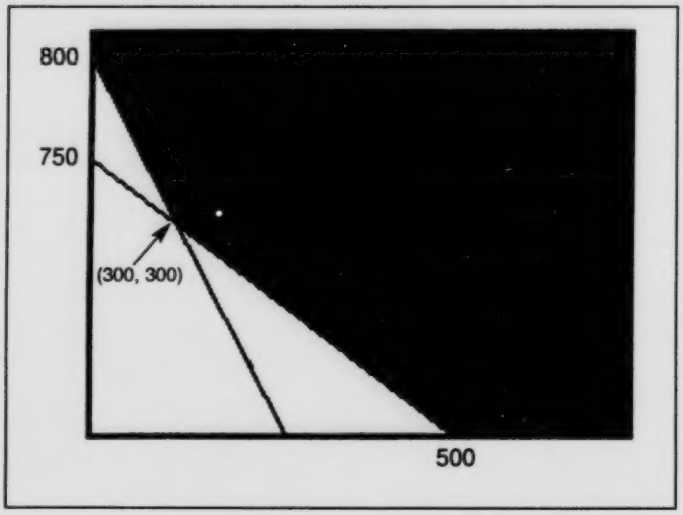
    $5x + 20y \geq 110$



Best solution is 14 hours for John, and 2 hours for Bill. Minimum cost is $143.00.

12. The question is more difficult than the rest.

$\frac{x}{15} + \frac{y}{10} \geq 50$ $\frac{x}{12} + \frac{y}{20} \geq 40$



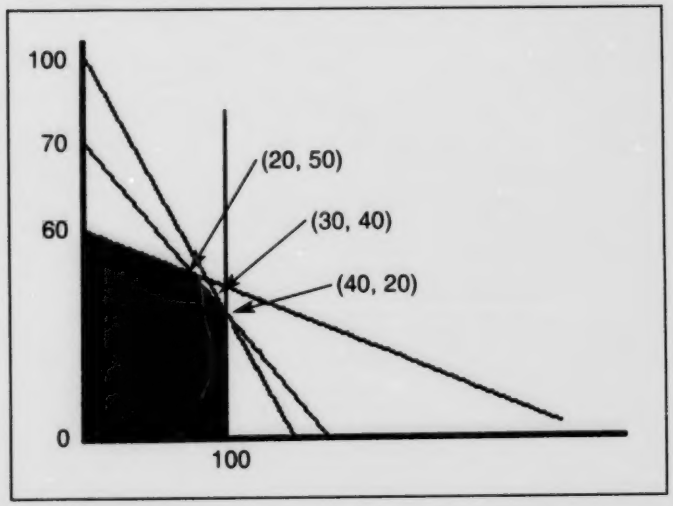
Best solution is 5 hours each. Minimum cost is $100.

Exercise 5: Solving Linear Programming Problems — Solutions (continued)

13. This problem involves students producing a report to a plant supervisor. They should produce a quality graph and a covering memo. This reinforces the technical communication aspect of the course. The graph could be drawn on a graphing program or painting program.

$3x + 3y \leq 210$ $\quad\quad$ $x \leq 40$

$4x + 2y \leq 200$ $\quad\quad$ $x + 2y \leq 120$



Best solution is at 20 of Product A and 50 of Product B for a profit of $1900.00.

Attachment D-1
Teaching Notes

Linear Inequalities

Notation

$>$	greater than	e.g., $10 > 6$	
$<$	less than	e.g., $6 < 10$	
$\geq$	greater than or equal to	e.g., $10 \geq 6$	$6 \geq 6$
$\leq$	less than or equal to	e.g., $6 \leq 10$	$6 \leq 6$

A mathematical sentence such as $5 + 8x > 16$ or $7 - 5x < 27$ is called a linear inequality in one variable (x in this case). Solving linear inequalities involves using the properties of solving equations with one exception: when you need to multiply or divide by a negative number, the sign of the inequality changes from $>$ to $<$ and vice versa.

Example 1

Since $-8 < 6$ Multiply both sides by -2
$-2(8) < -2(6)$
$16 < -12$ Not true
$16 > -12$ True

Example 2

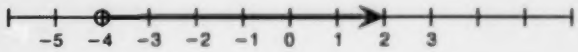
Solve for x: $-6 - 3x > 9$
$-3x > 15$ Sign does not change
$x < -5$ Sign does change

You can express your answer to an inequality on a number line.

Example 3

Solve for x and graph your answer.

$8 - 3x < 2x + 28$
$-5x < 20$
$x > -4$

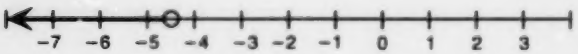


Notice that an open circle at 4 shows *greater than* but ***not*** *equal* to 4.

Example 4

Solve and graph:

$3x - 2 > 5x + 7$
$-2x > 9$
$x < \frac{-9}{2}$ or $x < -4\frac{1}{2}$

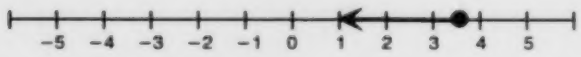


Linear Inequalities (continued)

Example 5

Solve and graph:

$$\begin{aligned} 3(x - 2x) &\geq 4x - 27 \\ 3x - 6x &\geq 4x - 27 \\ -3x - 4x &\geq -27 \\ -7x &\geq -27 \\ x &\leq \frac{-27}{-7} \\ x &\leq 3.86 \end{aligned}$$



Graphing Inequalities in a Plane

The general form of a linear inequality in 2 variables is $Ax + By < C$ or $Ax + By > C$.

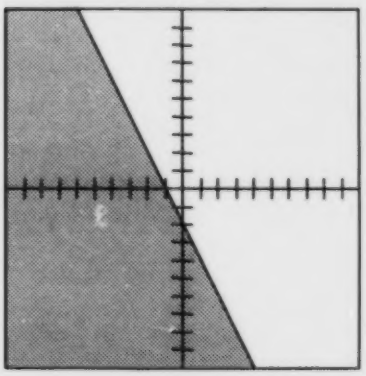
To graph an inequality like $3x + 5y > 6$, treat it like an equation, and rearrange to put it into slope-intercept form ($y = mx + b$). Recall that m is the slope and b is the y-intercept.

Steps:

1. Solve for y.
2. Draw the line as if the > or < was a =.
   Note: Use a solid line if you have a ≥ or ≤; use a dashed line if you have a > or <.
3. If > or ≥ shade above line.
   If < or ≤ shade below line.

Example 1

$-x - 2y \geq 4$
$-2y \geq x + 4$
$y \leq -2x - 2$
$m = -2, b = -2$

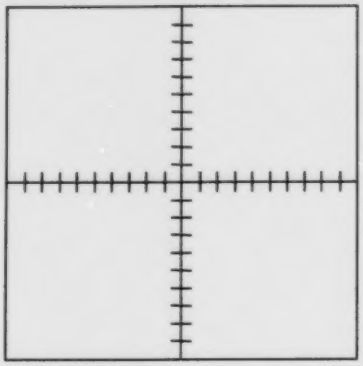


Graphing Inequalities in a Plane (continued)

Try the following examples on a graphing calculator, and sketch your solutions.

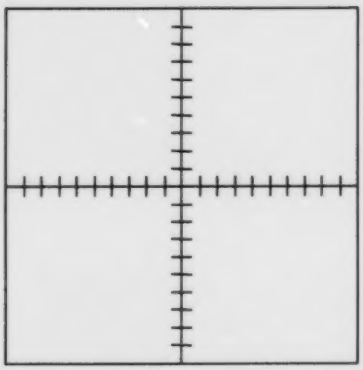
Example 2

$y + 2 < 0$
$y < -2$
$m = 0, b = -2$



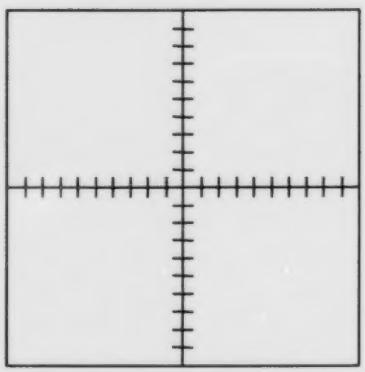
Example 3

$x + y > -1$



Example 4

$2x - y < 3$



Graphing and Solving Systems of Linear Inequalities

When you must solve a system of inequalities, you graph them both on the same set of axes. A system of equations has an implied "and" between them. Generally, you will deal with only two inequalities, but linear programming problems may deal with more than two.

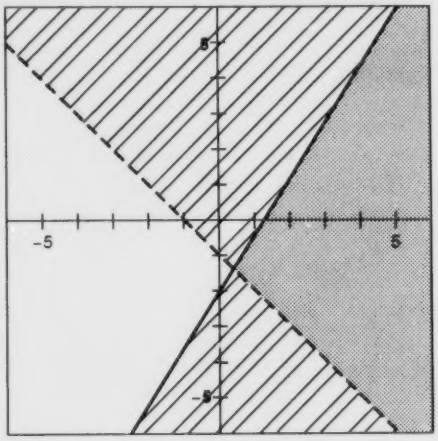
Example 1

Graph and solve. Give examples of possible solution points.

$$x + y > -1$$
$$3x - 2y > 4$$

$$x + y > -1$$
$$y > -x - 1$$
$$m = -1, b = -1$$

$$3x - 2y > 4$$
$$-2y > -3x + 4$$
$$y < \frac{3}{2}x - 2$$



Possible solution points are {(,) (,) (,)}

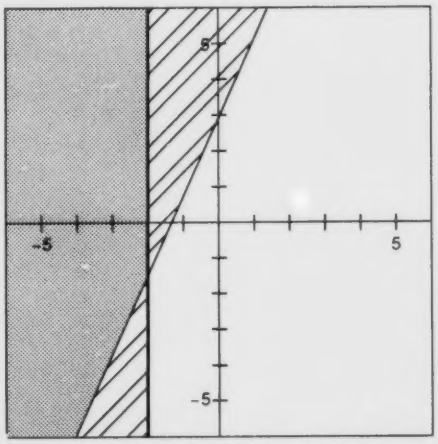
Example 2

Graph and solve.

$$2x - y \leq 3$$
$$x \leq -2$$

$$2x - y \leq -3$$
$$-y \leq -2x - 3$$
$$y \geq 2x + 3$$
$$m = 2, b = 3$$

$$x \leq -2$$



Word Problems — Setting Up Inequalities

Setting up a system of inequalities is no different than setting up a system of equations. However, you may have to deal with phrases like "at least" or "at most" in a system of inequalities.

Example 1

A pet-store farmer has turtles and rabbits. He has room for only 100 pets. He can sell turtles for \$10 each and rabbits for \$30 each. He decides that he must earn at least \$1500 to make the job worthwhile. What are some possible solutions for the farmer?

Solution

Use the variables as follows: x = turtles, y = rabbits

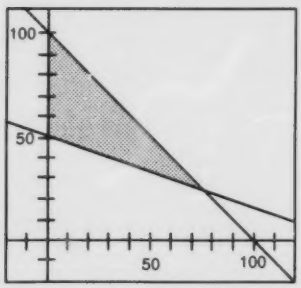
If he can have no more than 100 pets: $x + y \leq 100$

The profit inequality: $10x + 30y \geq 1500$

Rearrange these inequalities: $y \leq -x + 100$

$$y \geq -\frac{1}{3}x + 50$$

Resulting graph for these equations (solutions include any points in the shaded area).



Give two possible solutions for the farmer (solutions include any point in the shaded area).

Example 2

A rectangle has a length less than twice its width, its perimeter is at most 48, and its area is at least 32. Find the rectangle's dimensions.

Solution

Let x = width, y = length

Inequalities: $y < 2x$

$2x + 2y \leq 48$

$xy \geq 32$

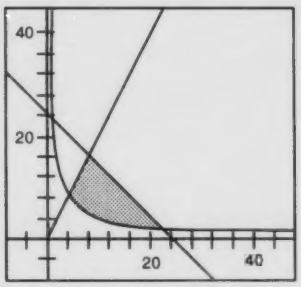
Rearrange: $y < 2x$

$y \leq -x + 24$

$$y > \frac{32}{x}$$

Word Problems — Setting Up Inequalities (continued)

Resulting graph:



Give two possible solutions for the dimensions of the rectangle. Solutions include any points in the shaded area (e.g., L = 12, W = 8).

Unit E
Budgets and Investments

Teachers are advised to refer to *Senior 3 Applied Mathematics: A Foundation for Implementation* as their ***primary*** source of suggestions for instruction and assessment. The following exercises are an excellent resource if supplementary material is required.

Exercise 1: Budgeting

Prepare a budget spreadsheet template that is based on the Monthly Budget Form illustrated in Attachment E-1. Use this budget template to answer the questions that follow.

1. Craig Bennet earns a net weekly salary of $645.25. The family receives a monthly Child Tax Benefit which amounts to $42.50 **per child** for each of their two children. His wife is a homemaker and receives no income.

   Monthly Expenses for the family include

a) monthly mortgage payment	$625.00
b) monthly car payment	223.50
c) average monthly telephone bill	17.40
d) average monthly hydro bill	120.00
e) food (average per month)	675.00
f) gasoline per month	180.00
g) clothing expenses	60.00
h) average car maintenance per month	30.00
i) entertainment per month	260.00
j) newspapers and periodicals	15.00

   Annual Expenses for the family include

a) yearly life insurance premium	480.00
b) special occasion spending per year	700.00
c) yearly car insurance premium	620.00
d) property taxes for the year	925.00
e) yearly home insurance premium	275.00
f) annual vacation	1800.00

   Prepare an estimated monthly budget for the Bennet family.

Exercise 1: Budgeting (continued)

2. Gerry and Terry Spivak are both employed. Terry receives a weekly salary of $391.82 after deductions. Gerry receives a net weekly salary of $381.42. The family receives a monthly Child Tax Benefit that amounts to $107.72.

Monthly Expenses for the family include

a) monthly first mortgage payment	$531.50
b) monthly second mortgage payment	201.65
c) monthly car payment	237.75
d) average monthly telephone bill	20.20
e) food (average per month)	740.00
f) gasoline per month	140.00
g) average monthly hydro bill	200.00
h) entertainment per month	160.00
i) average monthly credit card payment	200.00
j) car maintenance for the month	40.00
k) clothing	100.00

Annual Expenses for the family include

a) saving for summer vacation	2160.00
b) periodicals (per year)	102.00
c) yearly car insurance premium	520.00
d) yearly life insurance premium	362.40
e) home is assessed for property tax purposes at $20 000, the mill rate is 98 mills	
f) home insurance based on a home value of $60 000 at a cost of $0.42 per $100	

Prepare an estimated monthly budget for the Spivak family.

Exercise 1: Budgeting (continued)

3. Janice and Trevor Rees are both employed. Janice receives a weekly salary of $301.60 after deductions. Trevor nets $310.50 per week. The family receives a monthly Child Tax Benefit that amounts to $26.93 per child for each of their two children.

Monthly Expenses for the family include

a) monthly mortgage payment	$525.60
b) monthly car payment	186.40
c) average monthly telephone bill	18.60
d) average monthly hydro bill	225.00
e) gasoline per month	100.00
f) food (average per month)	525.00
g) clothing expenses for the month	55.00
h) average car maintenance for the month	50.00
i) average monthly credit card payment	200.00
j) child care (average per month)	200.00
k) monthly boat payment	130.00
l) entertainment per month	50.00

Annual Expenses for the family include

a) yearly car insurance premium	720.00
b) yearly vacation	940.00
c) special occasion spending per year	850.00
d) annual life insurance premium	240.00
e) home is assessed for property tax purposes at $30 000, the mill rate is 61 mills	
f) home insurance based on a home value of $50 000 at a cost of $0.62 per $100	

Prepare an estimated monthly budget for the Rees family.

Exercise 1: Budgeting (continued)

4. Natalie Hill earns $445.75 net per week. Her husband Tony earns $337.75 per week. The family receives a monthly Child Tax Benefit that amounts to $36.75 **per child** for each of their two children.

Monthly Expenses for the family include

a) monthly mortgage payment	$715.40
b) monthly car payment	206.10
c) average monthly telephone bill	23.00
d) total utilities for the month	305.00
e) monthly boat payment	230.00
f) food (average per month)	560.00
g) clothing expenses for the month	70.00
h) average car maintenance per month	35.00
i) average monthly credit card payment	200.00
j) gasoline per month	120.00
k) child care (average per month)	150.00
l) entertainment per month	250.00

Annual Expenses for the family include

a) yearly car insurance premium	610.00
b) home insurance (yearly premium)	660.00
c) yearly vacation	2000.00
d) special occasion spending for year	500.00
e) home is assessed for property tax purposes at $40 000, the mill rate is 60 mill	

a) Prepare an estimated monthly budget for the Hill family.

b) As the financial advisor for the Hill family, you notice that the Hills have no life insurance. Upon questioning Mr. Hill about this, he states, "I am not too worried about this yet; I am still a young healthy man." Explain to Mr. Hill why this logic is faulty and could prove to be a financial disaster for his young family.

c) The Hills are a little concerned about the family's present financial position. Mr. Hill suggests they reduce their amount of savings to balance the budget. Suggest to the Hills some areas in which you feel they could reduce spending and still balance their budget.

Exercise 1: Budgeting (continued)

5. Bob Sampson has a monthly take-home pay of $2135.00. His wife Elaine has a monthly net income of $2347.00. The Sampsons have no children.

Monthly Expenses for the Sampsons include

a) monthly mortgage payment	$900.00
b) monthly car payment	350.00
c) average monthly telephone bill	35.00
d) average monthly hydro bill	80.00
e) average monthly natural gas bill	75.00
f) monthly payment into an RRSP	150.00
g) food (average per month)	450.00
h) clothing expenses	83.00
i) car maintenance (average for the month)	50.00
j) gasoline per month	150.00
k) average monthly credit card payment	300.00
l) entertainment	400.00
m) newspapers and periodicals	30.00

Annual Expenses for the family include

a) yearly vacation	3000.00
b) special occasion spending per year	1200.00
c) yearly car insurance premium	550.00
d) home insurance (yearly premium)	300.00
e) home is assessed for property tax purposes at $50 000, the mill rate is 70 mills	

Use the template you created to answer the following questions.

Note: Each question requires you to go back to your original template for the family.

a) The Sampsons are considering having children. If they do, Mrs. Sampson is thinking about working part-time. Her monthly net income would drop to $1500 per month. What would be the amount available for other savings or expenditures (see under Summary, No. 5) if she were to do this?

b) If the Sampsons were to reduce their monthly spending on entertainment from $400 per month to $200 per month, what would be the amount available for other savings or expenditures (see under Summary, No. 5)?

c) The Sampson's tax advisor suggests that they place $400.00 a month into their RRSPs. The advisor reminds them that this money is a tax deduction and that the interest earned on this account is tax-free. If the Sampsons were to do this, what would be their new amount available for other savings or expenditures?

d) Mr. Sampson believes that they should forego their annual winter vacation for 1 year. If they were to do this, what would be the amount available for other savings or expenditures?

e) Mrs. Sampson says that in 4 years the mortgage on their home will be paid off and then they will really be able to save money in RRSPs. What amount will be available for other savings or expenditures if their mortgage was paid off but they decide to set aside $250.00 per month in a special account to cover future renovations and upkeep for their house?

Exercise 1: Budgeting (continued)

6. Use your prepared budget template to complete the following:

   Prepare an estimate of the amount you think your family or another family spends on the various categories on the budget form.

   Prepare a written report of your findings using the following guidelines.

   a) Why does the family you have chosen budget or not budget?

   b) What do you see as the main benefits of budgeting?

   c) What are the main obstacles for effective budgeting for this family?

   d) What recommendations would you make to someone who wants to prepare a budget for the first time?

   e) Summarize the strengths and weaknesses that you discovered in financial planning of this family.

7. In this exercise, you will examine the costs of living on your own. Check the newspapers to determine rent levels or the cost of purchasing a home.

   Estimate the various spending levels for those items you believe are necessary for your particular situation.

   Use your spreadsheet template to examine various "what if" scenarios such as the increase or decrease in income or spending levels, or perhaps the sharing of some expenses with a roommate.

   Prepare a written report documenting your findings, along with the completed budget forms for the different situations examined. Your numbers should be as realistic as possible for a typical person moving out on his/her own for the first time.

Exercise 2: Interest

1. Harland's GIC pays him 6% simple interest. How much interest will he earn on $4000 deposited in this account for 1 year? What will be his balance in this account at the end of the year?

2. The account in which Jane deposits her money pays 4.25% simple interest annually. How much interest will she earn on a deposit of $5000 left in the account for 6 months? What will be her balance in the account at the end of this time?

3. Malcolm has earned $24.00 in interest on $400 that he deposited in his account 1 year ago. What rate of interest was paid on this account?

4. A person was thinking of purchasing a leather couch and loveseat for $4000. The salesperson said that $200 would be "knocked off" the price of the bill if the amount was paid in cash right away instead of buying on the installment plan. If the purchaser knows that she will receive her income tax refund in 30 days, calculate the amount the person would gain by borrowing the money for 30 days at 9% to pay the bill.

5. A person borrowed $8000 from a business associate. Four months later the loan was repaid with a cheque for $8400. What rate of interest was the person charged?

6. A high school student wished to purchase an "old beater." The student took out a personal loan for 10 months in order to purchase a used vehicle. The interest rate was 15% per annum. If the student paid $268.75 in interest on the loan, what must have been the amount of the loan? Why might a student be forced to pay a higher rate of interest than a middle-aged person?

7. How many months will it take for a $2000 deposit to earn $120 interest at 18% per annum?

8. Jackson has a previous balance of $800 on his credit card statement. Interest was calculated on this balance at the rate 1.25% **per month**.
   a) Calculate his ending balance on the credit card.
   b) What must be his minimum monthly credit card payment if it must correspond to at least 5% of the ending balance or $10, whichever is greater?

9. You signed a promissory note for $3500 to pay your property taxes. The note was signed for 120 days. Interest was charged at the rate of 10.5% per year. Calculate the total amount due in 120 days.

10. Audrey has a previous balance of $600 on her credit card statement. Interest was calculated on this balance at the rate 1.30% **per month**.
    a) Calculate the interest charge on the previous balance.
    b) During the month she made additional purchases totalling $315.00. Calculate the new ending balance.
    c) What must be her minimum monthly credit card payment if it must correspond to at least 5% of the ending balance or $10, whichever is greater?

Exercise 2: Interest (continued)

11. Most people prefer to repay loans, such as car loans, by using equal monthly payments. On the next page is a chart showing the amount of each payment required to pay back a loan over a chosen period of time. For example, if you wished to pay off a $500 loan at 10% for 6 months you would have a monthly payment of

$$\$500 \times 0.17156 = \$85.78$$

Use this table to answer the following questions:

a) Calculate the monthly payment to repay a loan of $2500 at 12.5% for 18 months.

b) Calculate the monthly payment to repay a loan of $2000 at 9.5% over a period of 24 months.

c) Two people were comparing the total cost of repaying a $5000 loan at 12%. One person chose to pay the loan back in 12 months while the other person chose 36 months.

i) Calculate each person's monthly payment.

ii) Calculate the total paid back by each individual.

iii) Calculate the total difference paid back by the two individuals.

iv) Draw a conclusion about the total amount repaid and the length of time required to pay off a loan.

12. You are to research the cost of purchasing a used car when the purchase must be done through a lending agency such as a bank or credit union. You should investigate obtaining a car loan using the following guidelines:

a) Research the cost of purchasing the vehicle through the classified section of a newspaper or a magazine that displays and sells used cars.

b) Fill in a credit application from the lending agency.

c) Compare interest rates charged by various lending agencies.

d) Check the monthly payments required over different amortization periods (an amortization period is the time required to pay back the loan).

Exercise 2: Interest (continued)

Monthly Payment per $1 Borrowed						
Annual Rate	Number of Equal Monthly Payments					
	6	12	18	24	30	36
8%	0.17058	0.08699	0.05914	0.04523	0.03689	0.03134
8.5%	0.17082	0.08722	0.05837	0.04546	0.03712	0.03157
9%	0.17107	0.08745	0.05960	0.04568	0.03735	0.03180
9.5%	0.17132	0.08768	0.05983	0.04591	0.03758	0.03203
10%	0.17156	0.08792	0.06006	0.04614	0.03781	0.03227
10.5%	0.17181	0.08815	0.06029	0.04638	0.03804	0.03250
11%	0.17205	0.08838	0.06052	0.04661	0.03838	0.03274
11.5%	0.17230	0.08862	0.06075	0.04684	0.03851	0.03298
12%	0.17255	0.08885	0.06098	0.04707	0.03875	0.03321
12.5%	0.17280	0.08908	0.06121	0.04731	0.03898	0.03345
13%	0.17304	0.08932	0.06145	0.04754	0.03922	0.03369
13.5%	0.17329	0.08955	0.06168	0.04778	0.03946	0.03394
14%	0.17354	0.08979	0.06192	0.04801	0.03970	0.03418
14.5%	0.17379	0.09002	0.06215	0.04825	0.04994	0.03442
15%	0.17403	0.09026	0.06238	0.04849	0.04018	0.03467
15.5%	0.17428	0.09049	0.06262	0.04872	0.04042	0.03491
16%	0.17453	0.09073	0.06286	0.04896	0.04066	0.03516
16.5%	0.17478	0.09097	0.06309	0.04920	0.04091	0.03540
17%	0.17503	0.09120	0.06333	0.04944	0.04115	0.03565
17.5%	0.17528	0.09144	0.06357	0.04968	0.04139	0.03590
18%	0.17553	0.09168	0.06381	0.04992	0.04164	0.03615
18.5%	0.17577	0.09192	0.06404	0.05017	0.04189	0.03640
19%	0.17602	0.09216	0.06428	0.05041	0.04213	0.03666
19.5%	0.17627	0.09240	0.06452	0.05065	0.04238	0.03691
20%	0.17652	0.09263	0.06476	0.05090	0.04263	0.03716
20.5%	0.17677	0.09287	0.06500	0.05114	0.04288	0.03742
21%	0.17702	0.09311	0.06524	0.05139	0.04313	0.03768
21.5%	0.17727	0.09335	0.06549	0.05163	0.04338	0.03793
22%	0.17752	0.09359	0.06573	0.05188	0.04363	0.03813

Exercise 3: Compound Interest

1. An amount of $5000 is to be invested at 10% for 4 years. Find the final amount if the interest on the $5000 investment were compounded
   a) annually
   b) semi-annually
   c) quarterly

   Compare your answers for a, b, and c. Make a generalization about the amount of interest earned and the frequency at which the interest is compounded.

2. Susan would like to deposit her income tax refund in an account earning 5.4% interest, compounded monthly. She wants to withdraw her money at the end of 5 months. If her income tax refund was $389.00, what will be her balance at that time?

3. A depositor had $10 000 on deposit in a bank that pays interest at a rate of 5% a year, compounded semi-annually. How much more interest would the depositor have earned during the first year if the bank compounded interest quarterly rather than semi-annually?

4. A grandparent of a newborn child decided to invest $5000 in a GIC that pays interest at the rate of 6% compounded semi-annually. What compound amount would the child receive at age 21 (assume a full 21 years)?

5. A person borrowed $2000. The person suddenly fell ill and the lender agreed to forego any payment until the person was better. The lender charged 1.5% interest per month on the amount outstanding. The person was unable to make any payments for a full 2 years. What compound amount would now be owed to the lender when payments began again?

6. Compound interest is an excellent topic to cover using a spreadsheet. Create a spreadsheet template that would allow the person to enter the original principal, the interest rate per period and the number of periods. Your spreadsheet should automatically calculate the amount of the investment. Use the spreadsheet to perform numerical calculations involving the following "what if" scenarios.
   a) Use an original investment of $10 000 at 5% for 5 years.
   b) What happens to the original investment if the interest were to rise to 7%?
   c) How long would it take for the original investment at 5% to amount to $25 000?
   d) How long would it take for the original investment at 7% to amount to $25 000?

Exercise 4: Exponential Data

1. The growth of the value of a $7000 RRSP is as follows:

Time (years)	Value ($)	Time (years)	Value ($)
0	7 000	13	21 461
1	7 630	14	23 392
2	8 317	15	25 497
3	9 065	16	27 792
4	9 881	17	30 293
5	10 770	18	33 599
6	11 740	19	35 992
7	12 796	20	39 231
8	13 948	21	42 762
9	15 203	22	46 610
10	16 572	23	50 805
11	18 063	24	55 378
12	19 689	25	60 362

Plot these data, using a spreadsheet or a graphing calculator. Estimate the time needed for the RRSP to reach $100 000. Determine the estimated value of the RRSP after 50 years.

2. Inflation is one reason the prices of consumer goods increase. For example, a golf set that might cost $300 at one time, might cost $720 ten years later. The formula below shows the inflated prices of goods to a constant inflation percentage.

$$C = P(1 + i)^n$$

where C is the current price after n years of constant inflation

P is the price at the start of the time period

i is the annual inflation rate, as a decimal

n is the number of years

A car currently costs $15 000. Plot the expected cost of the vehicle over the next 10 years if the estimated annual rate of inflation is 3%.

3. You won $10 000 in a mini-lottery. Concerned that you would "blow" the money you decided to invest the money. A friend advised you to invest in an interest-bearing account that paid 6% interest compounded semi-annually. The compound amount of your investment could be found using the formula

$$A = P(1 + i)^n$$

where A = compound amount i = interest rate per period

P = original amount invested n = number of interest periods

Using a calculator or spreadsheet, determine the compound amount of your investment from years 1 to 10. Plot these data on a graph.

Later, you were distressed to find you could have earned 8% interest compounded annually on another investment. Plot the compound amount of this investment on the same graph as the 6% investment.

What difference is there in the compound amounts after 10 years?

Attachment E-1

MONTHLY BUDGET FORM

AVERAGE MONTHLY INCOME

		Weekly Amount	Annual Amount	
1)	***NET INCOME***			1) $_____
	Primary Income	$_____	$_____	
	Secondary Income	$_____	$_____	
	Other Income	$_____	$_____	
	Total Annual Income		$_____	
2)	***SAVINGS*** **(10% of Average Monthly Income)**			2) $_____
3)	***MONTHLY EXPENSES***			
	Mortgage or Rent		$_____	
	Car Payments		$_____	
	Telephone		$_____	
	Hydro		$_____	
	Other Utilities		$_____	
	Groceries		$_____	
	Clothing		$_____	
	Car Maintenance		$_____	
	Gasoline		$_____	
	Credit Card Debt		$_____	
	Entertainment		$_____	*MONTHLY*
	Other		$_____	*TOTAL*
	Other		$_____	
	TOTAL MONTHLY EXPENSES			3) $_____

		Annual Amount	Monthly Payment	
4)	***ANNUAL EXPENSES***			
	Car Insurance	$_____	$_____	
	Life Insurance	$_____	$_____	
	Property Taxes	$_____	$_____	
	Home Insurance	$_____	$_____	
	Vacations	$_____	$_____	*MONTHLY*
	Other	$_____	$_____	*TOTAL*
	TOTAL MONTHLY PAYMENTS			4) $_____

SUMMARY

1. Average Monthly Income			1) $_____
2. Savings	2)	$_____	
3. Total Monthly Expenses	3)	$_____	
4. Total Monthly Payments	4)	$_____	
Total amounts 2 + 3 + 4			$_____
5. Amount Available for Other Savings or Expenditures (Deficit)			5) $_____

Note: *If the person is in a deficit position, the budget needs to be analyzed for possible adjustments.*

Unit E
Budgets and Investments
Solutions

Exercise 1: Budgeting — Solutions

1.

MONTHLY BUDGET FORM

	Weekly Amount	Annual Amount		AVERAGE MONTHLY INCOME
1) *NET INCOME*				
Primary Income	$ 645.25	$ 33,553.00		
Secondary Income	$	$		
Other Income	$	$ 1,020.00	1)$	2,881.08
Total Annual Income		$ 34,573.00		
2) *SAVINGS* (10% of Average Monthly Income)			2)$	288.11
3) MONTHLY EXPENSES				
Mortgage or Rent		$ 625.00		
Car Payments		$ 223.50		
Telephone		$ 17.40		
Hydro		$ 120.00		
Other Utilities		$		
Groceries		$ 675.00		
Clothing		$ 60.00		
Car Maintenance		$ 30.00		
Gasoline		$ 180.00		
Credit Card Debt		$		
Entertainment		$ 260.00		
Other (Newspaper)		$ 15.00		*MONTHLY*
Other		$		*TOTAL*
TOTAL MONTHLY EXPENSES			3)$	2,205.90
	Annual Amount	**Monthly Payment**		
4) *ANNUAL EXPENSES*				
Car Insurance	$ 620.00	$ 51.67		
Life Insurance	$ 480.00	$ 40.00		
Property Taxes	$ 925.00	$ 77.08		
Home Insurance	$ 275.00	$ 22.92		
Vacations	$ 1800.00	$ 150.00		*MONTHLY*
Other (Christmas)	$ 700.00	$ 58.33		*TOTAL*
TOTAL MONTHLY PAYMENTS			4)$	400.00
SUMMARY				
1. Average Monthly Income			1)$	2,881.08
2. Savings		2)$ 288.11		
3. Total Monthly Expenses		3)$ 2,205.90		
4. Total Monthly Payments		4)$ 400.00		
Total Amounts 2 + 3 + 4			$	2,894.01
5. Amount Available for Other Savings or Expenditures (Deficit)			5)$	(12.92)

Note: *If the person is in a deficit position, the budget needs to be analyzed for possible adjustments.*

2.

			MONTHLY BUDGET FORM				
			Weekly		**Annual**		***AVERAGE***
1)	***NET INCOME***		**Amount**		**Amount**		***MONTHLY***
	Primary Income	$	391.82	$	20,374.64		***INCOME***
	Secondary Income	$	381.42	$	19,833.84		
	Other Income	$		$	1,292.64	1)$	3,458.43
	Total Annual Income			$	41,501.12		
2)	***SAVINGS*** **(10% of Average Monthly Income)**					2)$	345.84
3)	***MONTHLY EXPENSES***						
	Mortgage or Rent			$	733.15		
	Car Payments			$	237.75		
	Telephone			$	20.20		
	Hydro			$	200.00		
	Other Utilities			$			
	Groceries			$	740.00		
	Clothing			$	100.00		
	Car Maintenance			$	40.00		
	Gasoline			$	140.00		
	Credit Card Debt			$	200.00		
	Entertainment			$	160.00		
	Other			$			***MONTHLY***
	Other			$			***TOTAL***
	TOTAL MONTHLY EXPENSES					3)$	2,571.10
			Annual		**Monthly**		
4)	***ANNUAL EXPENSES***		**Amount**		**Payment**		
	Car Insurance	$	520.00	$	43.33		
	Life Insurance	$	362.40	$	30.20		
	Property Taxes	$	1960.00	$	163.33		
	Home Insurance	$	252.00	$	21.00		
	Vacations	$	2160.00	$	180.00		***MONTHLY***
	Other (Periodicals)	$	102.00	$	8.50		***TOTAL***
	TOTAL MONTHLY PAYMENTS					4)$	446.37
	SUMMARY						
	1. Average Monthly Income					1)$	3,458.43
	2. Savings			2)$	345.84		
	3. Total Monthly Expenses			3)$	2,571.10		
	4. Total Monthly Payments			4)$	446.37		
	Total Amounts 2 + 3 + 4					$	3,363.31
	5. Amount Available for Other Savings or Expenditures (Deficit)					5)$	95.12
	Note: *If the person is in a deficit position, the budget needs to be analyzed for possible adjustments.*						

3.

MONTHLY BUDGET FORM

		Weekly Amount	Annual Amount		AVERAGE MONTHLY INCOME
1)	**NET INCOME**				
	Primary Income	$ 310.50	$ 16,146.00		
	Secondary Income	$ 301.60	$ 15,683.20		
	Other Income	$	$ 646.32	1)$	2,706.29
	Total Annual Income		$ 32,475.52		
2)	**SAVINGS (10% of Average Monthly Income)**			2)$	270.63
3)	**MONTHLY EXPENSES**				
	Mortgage or Rent		$ 525.60		
	Car Payments		$ 186.40		
	Telephone		$ 18.60		
	Hydro		$ 225.00		
	Other Utilities		$		
	Groceries		$ 525.00		
	Clothing		$ 55.00		
	Car Maintenance		$ 50.00		
	Gasoline		$ 100.00		
	Credit Card Debt		$ 200.00		
	Entertainment		$ 50.00		
	Other (Child Care)		$ 200.00		MONTHLY
	Other (Boat Payment)		$ 130.00		TOTAL
	TOTAL MONTHLY EXPENSES			3)$	2,265.60
4)	**ANNUAL EXPENSES**	Annual Amount	Monthly Payment		
	Car Insurance	$ 720.00	$ 60.00		
	Life Insurance	$ 240.00	$ 20.00		
	Property Taxes	$ 1830.00	$ 152.50		
	Home Insurance	$ 310.00	$ 25.83		
	Vacations	$ 940.00	$ 78.33		MONTHLY
	Other (Christmas)	$ 850.00	$ 70.83		TOTAL
	TOTAL MONTHLY PAYMENTS			4)$	407.50
	SUMMARY				
	1. Average Monthly Income			1)$	2,706.29
	2. Savings		2)$ 270.63		
	3. Total Monthly Expenses		3)$ 2,265.60		
	4. Total Monthly Payments		4)$ 407.50		
	Total Amounts 2 + 3 + 4			$	2,943.73
	5. Amount Available for Other Savings or Expenditures (Deficit)			5)$	(237.44)

Note: *If the person is in a deficit position, the budget needs to be analyzed for possible adjustments.*

4.

MONTHLY BUDGET FORM

		Weekly Amount	Annual Amount		AVERAGE MONTHLY INCOME
1) **NET INCOME**					
	Primary Income	$ 445.75	$ 23,179.00		
	Secondary Income	$ 337.75	$ 17,563.00		
	Other Income	$	$ 882.00	1)$	3,468.67
	Total Annual Income		$ 41,624.00		
2) **SAVINGS (10% of Average Monthly Income)**				2)$	346.87
3) **MONTHLY EXPENSES**					
	Mortgage or Rent		$ 715.40		
	Car Payments		$ 206.10		
	Telephone		$ 23.00		
	Hydro		$ 305.20		
	Other Utilities		$		
	Groceries		$ 560.00		
	Clothing		$ 70.00		
	Car Maintenance		$ 35.00		
	Gasoline		$ 120.00		
	Credit Card Debt		$ 200.00		
	Entertainment		$ 250.00		
	Other (Child Care)		$ 150.00		MONTHLY
	Other (Boat Payment)		$ 230.00		TOTAL
	TOTAL MONTHLY EXPENSES			3)$	2,864.70
4) **ANNUAL EXPENSES**		Annual Amount	Monthly Payment		
	Car Insurance	$ 610.00	$ 50.83		
	Life Insurance	$ 0.00	$ 0.00		
	Property Taxes	$ 2400.00	$ 200.00		
	Home Insurance	$ 660.00	$ 55.00		
	Vacations	$ 2000.00	$ 166.67		MONTHLY
	Other (Christmas)	$ 500.00	$ 41.67		TOTAL
	TOTAL MONTHLY PAYMENTS			4)$	514.17
SUMMARY					
	1. Average Monthly Income			1)$	3,468.67
	2. Savings		2)$ 346.87		
	3. Total Monthly Expenses		3)$ 2,864.70		
	4. Total Monthly Payments		4)$ 514.17		
	Total Amounts 2 + 3 + 4			$	3,725.73
	5. Amount Available for Other Savings or Expenditures (Deficit)			5)$	(257.07)

Note: *If the person is in a deficit position, the budget needs to be analyzed for possible adjustments.*

b) It should be stressed to students that life insurance is an extremely important part of financial planning. Mr. Hill earns an annual net income of $23 179.00. Without this income, what would happen to his wife and children?

Emphasize to students that the purchase of life insurance at a relatively young age is usually not an expensive proposition. This could lead to a discussion of the cost of life insurance. A local life insurance broker could make a presentation to students on the benefits of purchasing life insurance.

Discuss the importance of Mrs. Hill purchasing an appropriate amount of life insurance. Her income totals $17 563.00 per year and the loss of this income plus the resulting increase in costs such as child care would create financial problems for the family.

c) Suggesting areas where spending could be cut is obviously a personal matter. Some possible areas that the Hill family might wish to examine carefully include:

- Entertainment at an average of $250.00 per month. Should the family examine some cheaper forms of entertainment?
- The boat payment of $230.00 per month is a large amount of money. Is the family getting full value for this money?
- Credit card debt is $200 per month. Can this amount be reduced?
- Total utilities of $305.20 per month seems high. Does the family home need to be re-insulated? Perhaps the family needs to examine other energy-saving ideas.
- The family might wish to forego a summer vacation this year to improve their financial position.

Stress to students that sometimes sacrifices need to be made in the short term. Putting off a particular purchase or form of entertainment can greatly improve one's long term financial position. Emphasize that budgeting is not a form of punishment but rather a guide for financial stability.

5.

MONTHLY BUDGET FORM

		Monthly Amount	Annual Amount		AVERAGE MONTHLY INCOME
1)	***NET INCOME***				
	Primary Income	$ 2,347.00	$ 28,164.00		
	Secondary Income	$ 2,135.00	$ 25,620.00		
	Other Income	$	$	1)$	4,482.00
	Total Annual Income		$ 53,784.00		
2)	***SAVINGS*** **(10% of Average Monthly Income)**			2)$	448.20
3)	***MONTHLY EXPENSES***				
	Mortgage or Rent		$ 900.00		
	Car Payments		$ 350.00		
	Telephone		$ 35.00		
	Hydro		$ 80.00		
	Other Utilities		$ 75.00		
	Groceries		$ 450.00		
	Clothing		$ 83.00		
	Car Maintenance		$ 50.00		
	Gasoline		$ 150.00		
	Credit Card Debt		$ 300.00		
	Entertainment		$ 400.00		
	Other (R.R.S.P.)		$ 150.00		MONTHLY
	Other (Newspapers)		$ 30.00		TOTAL
	TOTAL MONTHLY EXPENSES			3)$	3,053.00
4)	***ANNUAL EXPENSES***	Annual Amount	Monthly Payment		
	Car Insurance	$ 550.00	$ 45.83		
	Life Insurance	$ 0.00	$ 0.00		
	Property Taxes	$ 3500.00	$ 291.67		
	Home Insurance	$ 300.00	$ 25.00		
	Vacations	$ 3000.00	$ 250.00		MONTHLY
	Other (Christmas)	$ 1200.00	$ 100.00		TOTAL
	TOTAL MONTHLY PAYMENTS			4)$	712.50
	SUMMARY				
	1. Average Monthly Income			1)$	4,482.00
	2. Savings		2)$ 448.20		
	3. Total Monthly Expenses		3)$ 3,053.00		
	4. Total Monthly Payments		4)$ 712.50		
	Total Amounts 2 + 3 + 4			$	4,213.70
	5. Amount Available for Other Savings or Expenditures (Deficit)			5)$	268.30

Note: *If the person is in a deficit position, the budget needs to be analyzed for possible adjustments.*

a) If Mrs. Sampson works part-time and earns only $1500.00 per month, the family would be in a deficit position of $494.00. This type of decision would probably require the Sampsons to make a major lifestyle change.

b) If the Sampsons reduce spending on entertainment by $200.00, they would see the amount available for other savings or expenditures increase to $468.30.

c) Placing $400 per month into an RRSP requires the family to increase the current amount by $250.00 per month. As a result, they would see their amount available for other savings or expenditures decrease to $18.30 per month. This would be an excellent place to discuss the advantages of earning interest in a tax-free investment such as a RRSP. A financial planner might be willing to come into the class to discuss the whole area of RRSPs.

d) By not taking a vacation for one year, the Sampsons would be in surplus position of $518.30. This might lead to a discussion of the need to have recreatiional activities built into the budget. If family members feel that the budget is too restrictive, they will soon abandon it. A more realistic approach might be to examine a cheaper vacation rather than not taking one at all.

e) When the mortgage is paid off, the Sampsons would have $918.30 per month for additional savings or expenditures. Discuss the advantages of paying off a mortgage as quickly as possible. An interesting calculation for students is to calculate the total amount paid on a mortgage if they chose a 20-year amortization period versus a 15-year amortization period.

Exercise 2: Interest — Solutions

1. Interest: $I = prt$
   $I = 4000 \times 6\% \times 1$
   $I = \$240.00$

   New Balance: $\$4000 + 240 = \$4240.00$

2. Interest: $I = prt$

$$I = 5000 \times 4.25\% \times \frac{6}{12}$$

$$I = \$106.25$$

3. $I = prt$

$$r = \frac{I}{pt}$$

$$r = \frac{24}{400 \times 1}$$

$$r = \frac{24}{400}$$

$$r = 6\%$$

4. Amount required to be borrowed: $\$4000.00 - \$200.00 = \$3800.00$

   Interest: $I = prt$

$$I = 3800 \times 9\% \times \frac{30}{365}$$

$$I = \$28.11$$

   Amount Saved: $\$200.00 - \$28.11 = \$171.89$

5. Interest: $\$8400 - \$8000 = \$400$

$$I = prt$$

$$r = \frac{I}{pt}$$

$$r = \frac{400}{800 \times \frac{4}{12}}$$

$$r = \frac{400}{2666.67}$$

$$r = 15\%$$

Exercise 2: Interest — Solutions (continued)

6. $I = prt$

$$p = \frac{I}{rt}$$

$$p = \frac{268.75}{0.15 \times \frac{10}{12}}$$

$$p = \frac{268.75}{0.125}$$

$p = \$2150$

Discuss the concept of risk and the impact it has on interest rates.

Ask students if any of them have taken out a car loan. If they have, there is a good chance that they needed someone to co-sign the note.

7. $I = prt$

$$t = \frac{I}{pr}$$

$$t = \frac{120}{2000 \times 0.18}$$

$$t = \frac{120}{360}$$

$t = \frac{1}{3}$ of a year

This equals $12 \times \frac{1}{3} = 4$ months.

8. a) Interest: $800 x 1.25% = $10.00

Ending balance: $800 + $10 = $810.00

b) Minimum monthly payment: $810.00 x 5% = $40.50

9. Interest:

$I = prt$

$$I = 3500 \times 10.5\% \times \frac{120}{365}$$

$I = \$120.82$

Total amount due: $3500 + 120.82 = $3620.82

10. a) Interest: $600 x 1.30% = $7.80

b) Ending balance: $600.00 + 7.80 + 315.00 = $922.80

c) Minimum monthly payment: $922.80 x 5% = $46.14

Exercise 2: Interest — Solutions (continued)

11. a) $2500 x 0.06121 = $153.03

    b) $2000 x 0.04591 = $91.82

    c) i) Over 12 months: $5000 x 0.08885 = $444.25 per month

    Over 36 months: $5000 x 0.03321 = $166.05 per month

    ii) Total paid over 12 months: $444.25 x 12 = $5331.00

    Total paid over 36 months: $166.05 x 36 = $5977.80

    iii) Person choosing 36 months paid: $5977.80 – $5331.00 = $646.80 extra

    iv) Students should conclude that the longer you take to pay back a loan, the more interest you will pay.

    Lead a discussion of why people might have to choose the longer repayment period even though it is financially more expensive in the long run.

12. Answers will vary.

Exercise 3: Compound Interest — Solutions

1. a)

$A = P(1+i)^n$

$A = 5000(1+0.10)^4$

$A = 5000(1.10)^4$

$A = \$7320.50$

b)

$i = \frac{10\%}{2} = 5\% = 0.05$

$n = 4 \times 2 = 8$ times

$A = P(1+i)^n$

$A = 5000(1+0.05)^8$

$A = 5000(1.05)^8$

$A = \$7387.30$

c)

$i = \frac{10\%}{4} = 2.5\% = 0.025$

$n = 4 \times 4 = 16$ times

$A = P(1+i)^n$

$A = 5000(1+0.025)^{16}$

$A = 5000(1.025)^{16}$

$A = \$7422.55$

As shown in (a), (b), and (c), the more often your money is compounded in a year, the greater the total amount you will eventually receive. Stress that the impact of this is even greater over a long period of time.

2.

$i = \frac{5.4\%}{12} = 0.45\% = 0.0045$

$n = 5$ months

$A = P(1+i)^n$

$A = 389(1+0.0045)^5$

$A = 389(1.0045)^5$

$A = \$397.83$

3. Semi-annually

$i = \frac{5\%}{2} = 2.5\% = 0.025$

$n = 1 \times 2$ times

$A = P(1+i)^n$

$A = 10\,000(1+0.25)^2$

$A = 10\,000(1.025)^2$

$A = \$10\,506.25$

Quarterly

$i = \frac{5\%}{4} = 1.25\% = 0.0125$

$n = 1 \times 4 = 4$ times

$A = P(1+i)^n$

$A = 10\,000(1+0.0125)^4$

$A = 10\,000(1.0125)^4$

$A = \$10\,509.45$

Extra earned by investing quarterly: \$10 509.45 – \$10 506.25 = \$3.20.

Once again, it should be emphasized the longer that you do this the greater the impact on your final total.

4. $A = P(1 + r)^n$
   $= 5000\ (1 + 0.03)^{42}$
   $= \$17\,303.48$

5. $A = P(1 + r)^n$
   $= 2000\ (1 + 0.015)^{24}$
   $= \$2859.01$

6. a) Compound amount after 5 years at 5% = \$12 762.80.

   b) Compound amount after 5 years at 7% = \$14 025.50.
   Extra interest earned: \$14 025.50 – \$12 762.880 = \$1262.70

   c) The investment at 5% will reach \$25 000 between the 18th and 19th years.

   d) The investment at 7% will reach \$25 000 between the 13th and 14th years.

Unit F
Data Management and Analysis

Teachers are advised to refer to *Senior 3 Applied Mathematics: A Foundation for Implementation* as their ***primary*** source of suggestions for instruction and assessment. The following exercises are an excellent resource if supplementary material is required.

Definitions

Discrete variables: A discrete variable assumes only certain specified values. Discrete data are sometimes obtained by counting and, in these situations, the only scores are whole numbers, that is, 0, 1, 2, 3, 4, 5, . . . (e.g., the number of times a person does the dishes in a week or the number of kittens in a litter). In other cases, the scores may represent a maximum or minimum of some quantity during a specified time period. For example, the highest temperature reached each day in a specific week (see example below).

In a discrete distribution, values obtained by interpolation between the points representing collected data have no meaning.

Continuous variables: A continuous variable is one that can be any value between certain limits, and is limited only by the precision of the measuring instrument. A range is identified within which it is reasonable that the measured attribute varies continuously. The time taken to bring a given quantity of water to boiling point will be measured with an identifiable precision; for example, 2.43 minutes, correct to two decimal places. Variables such as height, weight, and temperature can be considered examples of continuous data.

In a continuous distribution, information may be obtained by interpolating from the graph of points representing the collected data.

Time series: A set of data displayed within a given time span.

Glyphs: Pictorial/graphic depiction of data.

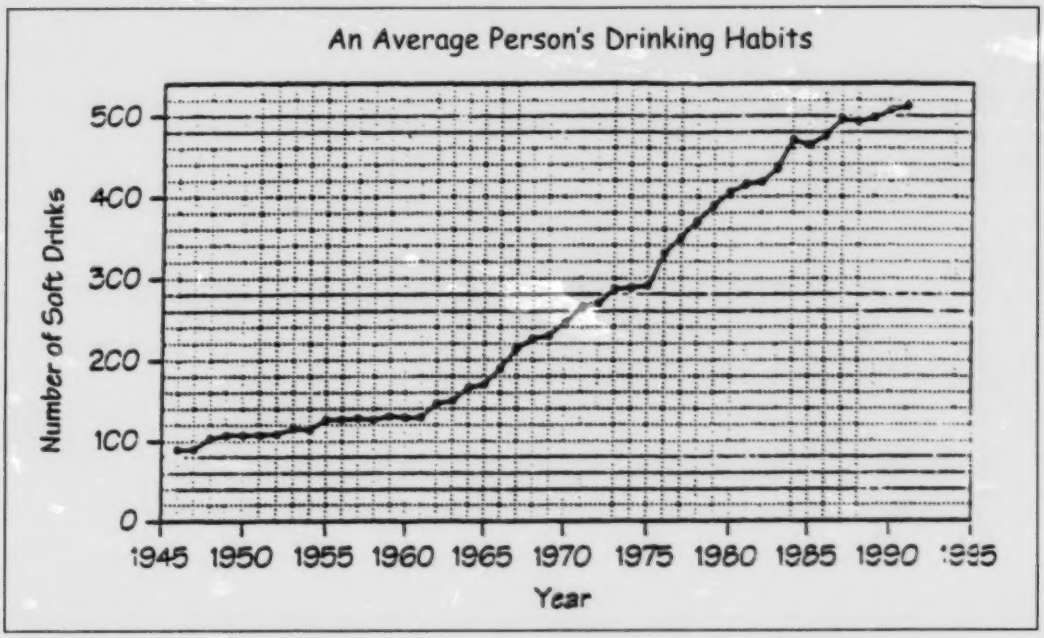
Contour lines: Lines representing data based on different elevations, weather, vegetation, natural resources, etc. (e.g., atmospheric or topographic maps).

Interpolation: Finding values within the range of the data.

Extrapolation: Finding values outside the range of the data.

Exercise 1: Time Series Plots

1. The following time series plot shows how many 12-ounce soft drinks the average person in the United States drank each year from 1945 to 1991.



a) About how many soft drinks did the average person drink in 1950? In 1970?

b) About how many six-packs of soft drinks did the average person drink in 1980?

c) About how many soft drinks did the average person drink per week in 1950? In 1980?

d) When did the average person start drinking one or more soft drinks per day? About how many soft drinks per day did people average in 1990?

e) If the recent trend continues, about how many soft drinks will the average person drink in the year 2000?

f) In what year did soft drink consumption start to "take off"? Hypothesize as to what caused this.

g) Who is the "average person"?

h) Write a summary of the trend in soft-drink consumption shown by the plot.

Exercise 1: Time Series Plots (continued)

2. The following table gives the median age at first marriage for men and women in the United States from 1900 to 1990 at 10-year increments.

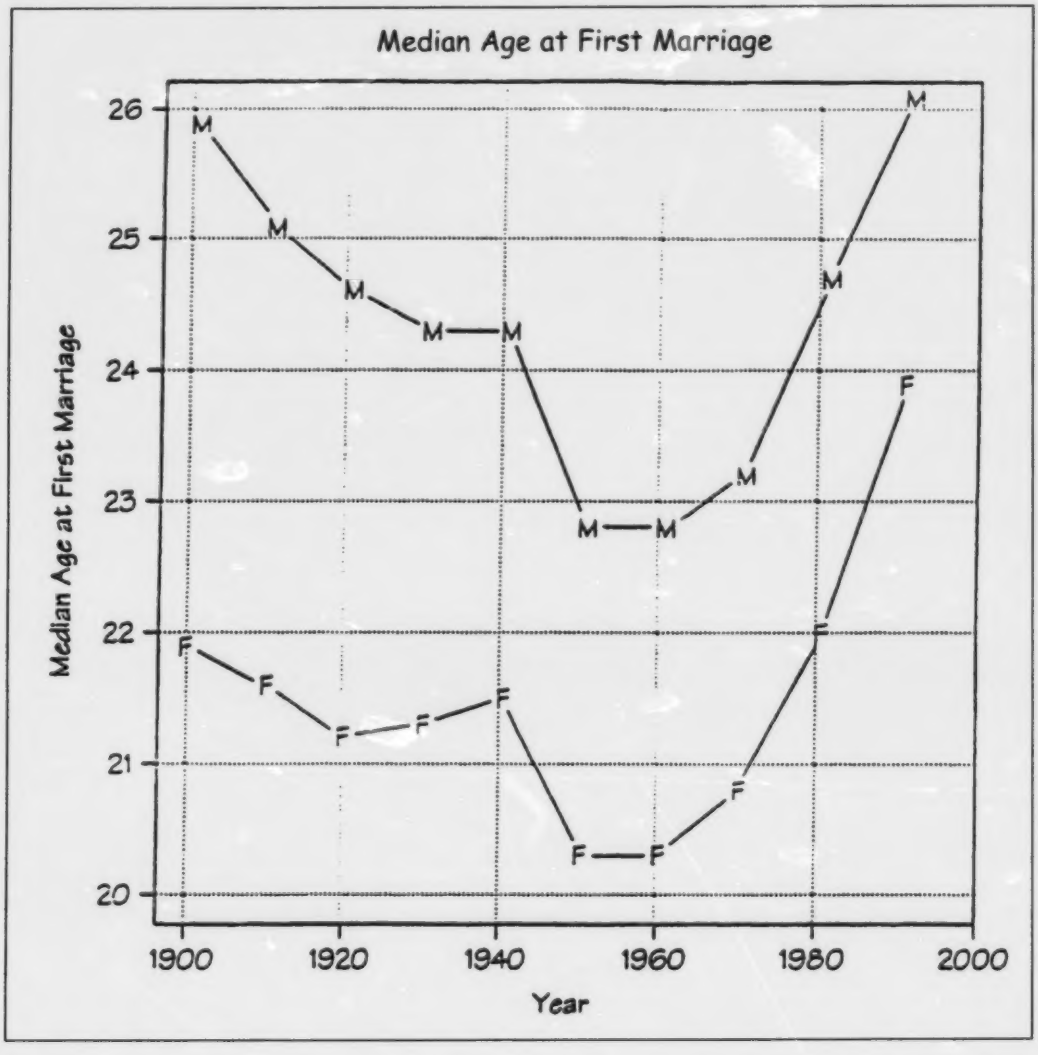
Median Age at First Marriage		
Year	Males	Females
1900	25.9	21.9
1910	25.1	21.6
1920	24.6	21.2
1930	24.3	21.3
1940	24.3	21.5
1950	22.8	20.3
1960	22.8	20.3
1970	23.2	20.8
1980	24.7	22.0
1990	26.1	23.9

a) Of the women married in 1900 for the first time, what percentage of them were 21.9 years old or younger? From these data, can you determine the percentage of men married in 1900 for the first time who were age 30 or older? What can you say about this percentage?

b) Most people report their ages as integers in years and months; for example you say that you are "16 years old," or possibly "16 years and 4 months." What does an age of 21.9 represent in terms of years and months?

Exercise 1: Time Series Plots (continued)

3. The data from question 2 are displayed in the graph below.



a) A similar overall pattern is seen in both male and female ages in this plot. Write a description of this pattern.

b) The largest decrease in the median age occurred during which decade for females? For males?

c) i) Draw a graph showing the difference in median ages of males and females at the time of first marriage from 1900 onward.

ii) Write a description of the pattern of age differences.

iii) Extrapolate from the original graph the ages at the time of first marriage for males and for females in the year 2000. Estimate the age difference in the year 2000.

Exercise 1: Time Series Plots (continued)

4. The following table lists the American League home run (HR) champions from 1921 to 1993.

Year	American League	HR	Year	American League	HR
1921	Babe Ruth, New York	59	1960	Mickey Mantle, New York	40
1922	Ken Williams, St. Louis	39	1961	Roger Maris, New York	61
1923	Babe Ruth, New York	41	1962	Harmon Killebrew, Minnesota	48
1924	Babe Ruth, New York	46	1963	Harmon Killebrew, Minnesota	45
1925	Bob Meusel, New York	33	1964	Harmon Killebrew, Minnesota	49
1926	Babe Ruth, New York	47	1965	Tony Conigliaaro, Boston	32
1927	Babe Ruth, New York	60	1966	Frank Robinson, Baltimore	49
1928	Babe Ruth, New York	54	1967	Carl Yastrzemsi, Boston	44
1929	Babe Ruth, New York	46		Harmon Killebrew, Minnesota	
1930	Babe Ruth, New York	49	1968	Frank Howard, Washington	44
1931	Babe Ruth, New York	46	1969	Harmon Killebrew, Minnesota	49
	Lou Gehrig, New York		1970	Frank Howard, Washington	44
1932	Jimmy Foxx, Philadelphia	58	1971	Bill Melton, Chicago	33
1933	Jimmy Foxx, Philadelphia	48	1972	Dick Allen, Chicago	37
1934	Lou Gehrig, New York	49	1973	Reggie Jackson, Oakland	32
1935	Jimmy Foxx, Philadelphia	36	1974	Dick Allen, Chicago	32
	Hank Greenberg, Detroit		1975	George Scott, Milwaukee	36
1936	Lou Gehrig, New York	49		Reggie Jackson, Oakland	
1937	Joe Dimaggio, New York	46	1976	Graig Nettles, New York	32
1938	Hank Greenberg, Detroit	58	1977	Jim Rice, Boston	39
1939	Jimmy Foxx, Boston	35	1978	Jim Rice, Boston	46
1940	Hank Greenberg, Detroit	41	1979	Gorman Thomas, Milwaukee	45
1941	Ted Williams, Boston	37	1980	Reggie Jackson, New York	41
1942	Ted Williams, Boston	36		Ben Oglivie, Milwaukee	
1943	Rudy York, Detroit	34	1981	Bobby Grich, California	22
1944	Nick Etten, New York	22		Tony Armas, Oakland	
1945	Vern Stephens, St. Louis	24		Dwight Evans, Boston	
1946	Hank Greenberg, Detroit	44		Eddie Murray, Baltimore	
1947	Ted Williams, Boston	32	1982	Gorman Thomas, Milwaukee	39
1948	Joe DiMaggio, New York	39		Reggie Jackson, California	
1949	Ted Williams, Boston	43	1983	Jim Rice, Boston	39
1950	Al Rosen, Cleveland	37	1984	Tony Armas, Boston	43
1951	Gus Zernial, Chicago-		1985	Darrell Evans, Detroit	40
	Philidelphia	33	1986	Jesse Barfield, Toronto	40
1952	Larry Doby, Cleveland	32	1987	Mark McGwire, Oakland	49
1953	Al Rosen, Cleveland	43	1988	Jose Canseco, Oakland	42
1954	Larry Doby, Cleveland	32	1989	Fred McGriff, Toronto	36
1955	Mickey Mantle, New York	37	1990	Cecil Fielder, Detroit	51
1956	Mickey Mantle, New York	52	1991	Jose Canseco, Oakland	44
1957	Roy Sievers, Washington	42		Cecil Fielder, Detroit	
1958	Mickey Mantle, New York	42	1992	Juan Gonzalez, Texas	43
1959	Rocky Colavito, Cleveland	42	1993	Juan Gonzalez, Texas	46
	Harmon Killebrew, Washington				

a) Create a scatterplot for this set of data.

Question 4: From *Exploring Data* by James M. Landwehr and Ann E. Watkins. ©1996 by AT&T Corp. Published by Dale Seymour Publications, an imprint of Pearson Learning. Used by permission.

Exercise 1: Time Series Plots (continued)

b) It is still difficult to see general trends, as the highs and lows of the graph are most noticeable. To remove the large fluctuations from the data and make it easier to see any trends, the *smoothing* method will be applied.

To find a smoothed value you take the data from the information from the year before and the year after and the current year, then find the median. For the first and last years of data, the original data are used.

Below is the smoothed version of the first twenty years of the home run champions' data.

The smoothed value for 1921 is the original value. For 1922, the home runs from 1921, 1922, and 1923 are used and the median of these three values becomes the smoothed value.

Year	Home Runs	Smoothed Values
1921	59	59
1922	39	41
1923	41	41
1924	46	41
1925	33	46
1926	47	47
1927	60	54
1928	54	54
1929	46	49
1930	49	46
1931	46	
1932		
1933		
1934		
1935		
1936		
1937		
1938		
1939		
1940		
1941		

c) Complete the smoothed value column through 1993 for the next 10 American League home run champions.

d) What happened around 1940 that could have affected the number of home runs hit?

e) Did the increase in the number of games from 154 to 162 in 1961 affect the number of home runs hit?

Exercise 1: Time Series Plots (continued)

f) Do you think any of the following rule changes affected the number of home runs hit? Why or why not?

— 1926: A ball hit over a fence that is less than 250 feet from home plate will not be counted as a home run.

— 1931: A fair ball that bounces over a fence will be counted as a double instead of a home run.

— 1959: New ball parks must have a minimum distance of 325 feet down the foul lines and 400 feet in centre field.

— 1969: The strike zone is decreased in size to include only the area from the armpit to the top of the knee.

— 1969: The pitcher's mound is lowered, giving the advantage to the hitter.

— 1971: All batters must wear helmets.

Should there be some indication in the original data about rule changes? Why or why not?

g) In 1981 there was a strike-shortened season. Is this evident in the original data? In the smoothed data?

h) The end points seem out of place in the smoothed data. What suggestions do you have for changing the way these points are displayed?

i) Imagine a curve through the smooth values. Try to predict the number of home runs hit in the year 2000.

j) Some people feel that smoothing is not a legitimate method. Choose a partner. One of you will be pro-smoothing and the other anti-smoothing. Write a summary of your arguments and which point of view you finally agreed on.

Exercise 1: Time Series Plots (continued)

5. The following table shows the winning heights for the high jump competitions in the 1928-1992 Olympics. The heights are given in feet and hundredths of a foot.

Year	Women Winner, Country	Height (feet)	Men Winner, Country	Height (feet)	Ratio
1928	Ethel Catherwood, Canada	5.21	Robert King, United States	6.38	0.817
1932	Jean Shiley, United States	5.44	Duncan McNaughton, Canada	6.46	0.842
1936	Ibolya Csak, Hungary	5.25	Cornelius Johnson, United States	6.67	0.787
1948	Alice Coachman, United States	5.50	John Winter, Australia	6.50	0.846
1952	Esther Brand, South Africa	5.48	Walter Davis, United States	6.71	0.817
1956	Mildred McDaniel, United States	5.77	Charles Dumas, United States	6.96	0.829
1960	Iolanda Balas, Romania	6.06	Robert Shavlakadze, USSR	7.08	0.856
1964	Iolanda Balas, Romania	6.23	Valery Brumel, USSR	7.15	0.871
1968	Miloslava Rezkova, Czechoslovakia	5.96	Dick Fosbury, United States	7.35	0.811
1972	Ulrike Meyfarth, West Germany	6.29	Yuri Tarmak, USSR	7.31	0.860
1976	Rosemarie Ackermann, East Germany	6.33	Jacek Wszola, Poland	7.38	
1980	Sara Simeoni, Italy	6.46	Gerd Wessig, East Germany	7.73	
1984	Ulrike Meyfarth, West Germany	6.63	Dietmar Moegenburg, West Germany	7.71	
1988	Louise Ritter, United States	6.67	Gennadiy Avdeyenko, USSR	7.81	
1992	Heike Henkel, Germany	6.63	Javier Sotomayor, Cuba	7.67	

Quickly scanning this table shows that the winning heights for both women and men have generally increased over the years, and that the men's heights are greater than the women's heights. In this application, you will explore questions concerning comparisons of women's heights to men's heights.

a) What was the first year in which the women's winning height was greater than the men's 1948 winning height?

b) Suppose a winning height is 6 feet 5 inches. How would this value be represented in the table? Express 7.81 feet in terms of feet and inches.

c) Find the 2 years when the Olympics were not held. Why were the Olympics not held in these years?

d) Why are the countries for the women's winners in 1972, 1976, and 1984 given as "West Germany" and "East Germany," while the country for the 1992 winning woman is listed simply as "Germany?"

e) To compare the women's and men's results, construct the ratio of the women's winning height to the men's winning height for each year, and then analyze these values. The first 10 ratios are filled in the previous table. Calculate the ratio of the women's winning height to the men's winning height for each of the remaining years.

f) Construct a plot over time with the year on the horizontal axis and the ratio of the women's winning height to the men's winning height on the vertical axis. (Do not connect the points.)

Question 5: From *Exploring Data* by James M. Landwehr and Ann E. Watkins. ©1996 by AT&T Corp. Published by Dale Seymour Publications, an imprint of Pearson Learning. Used by permission.

Exercise 1: Time Series Plots (continued)

g) Complete the following table of smoothed values by taking the median of the value for the year in question, the year before, and the year after. For example, to get the smoothed value for 1952, find the median of the values for 1948, 1952, and 1956.

Year	Ratio	Smoothed Values
1928	0.817	0.817
1932	0.842	0.817
1936	0.787	0.842
1948	0.846	
1952	0.817	
1956	0.829	
1960	0.856	
1964	0.871	
1968		
1972		
1976		
1980		
1984		
1988		
1992		

h) Construct a plot over time with the year on the horizontal axis and the smoothed values of the ratios on the vertical axis.

i) Comment on the results. How could you determine when the ratio is likely to reach the value of 1? What would that indicate?

Exercise 1: Time Series Plots (continued)

6. Use a time-plot graph to chart the changes in life expectancy at birth in Canada. Draw one line for females and one for males on the same graph.

Life Expectancy at Birth, by Sex, Canada				
Years	**Both Sexes**	**Males**	**Females**	**Difference**
1920-22	59.37	58.84	60.60	1.76
1930-32	61.00	60.00	62.06	2.06
1940-42	64.58	63.04	66.31	3.27
1950-52	68.51	66.40	70.90	4.50
1960-62	71.14	68.44	74.26	5.82
1970-72	72.74	69.40	76.45	7.05
1980-82	75.39	71.88	79.06	7.18
1990-92	77.80	74.61	80.97	6.36

Table: Adapted from Statistics Canada, "Life Expectancy at Birth, by Sex, Canada, Provinces," adapted from "Health Indicators," Catalogue No. 82-221.

Exercise 1: Time Series Plots (continued)

7. Use a time plot chart to graph the sales for Ontario, Quebec, Manitoba, and Saskatchewan. Draw one line for each province on the same graph.

Department Store Sales, Canada and the Provinces ($ thousands)				
Province	**1992**	**1993**	**1994**	**1995**
Canada	13 011 856	12 793 896	13 298 958	13 923 627
Newfoundland	180 480	180 996	201 955	x
Prince Edward Island	54 728	54 243	54 741	x
Nova Scotia	438 358	442 876	445 741	458 235
New Brunswick	297 524	297 759	328 130	344 271
Quebec	2 345 978	2 313 854	2 412 466	2 544 193
Ontario	5 396 775	5 385 283	5 586889	5 812 769
Manitoba	537 233	530 852	553 603	592 910
Saskatchewan	368 306	362 942	393 300	437 424
Alberta	1 434 761	1 370 812	1 423 183	1 522 220
British Columbia	1 957 710	1 854 278	1 898 982	1 945 934

x = confidential

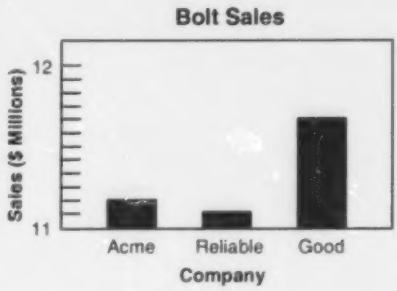
Table: Adapted from Statistics Canada, "Department Store Sales, Canada and the Provinces," adapted from the CANSIM Database, Matrix No. 112.

Exercise 2: Deceptive Graphs

If one or more parts of a graph are distorted, a deceptive graph can result. When reading a graph, check to see that scales are not condensed or expanded, that labels are truthful, and that data are plotted accurately.

1. The Good Bolt Company produced the following graph from the data comparing bolt sales of the three largest manufacturers.

Bolt Sales ($ millions)	
Acme	11.2
Reliable	11.1
Good	11.8

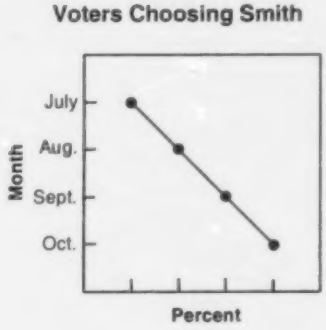


a) What impression is given by the graph? How has it been created?

b) Draw a graph to depict the data less deceptively.

2. Candidate Jones published this graph from the data showing the results of voter preference polls of candidate Smith.

Percentage of Voters Choosing Smith			
July	Aug.	Sept.	Oct.
20	30	40	50



a) According to the polls is Smith gaining or losing popularity?

b) What impression is given by the graph? Why?

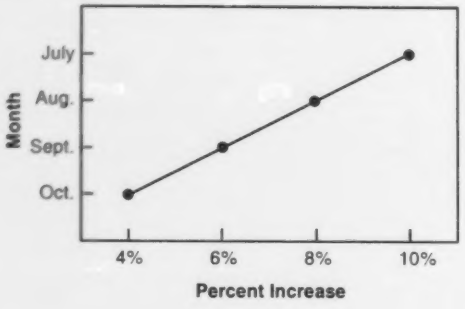
c) Redraw the graph to correctly reflect the data.

d) Describe three ways you can create a misleading impression using a bar graph. Describe two ways you can create a misleading impression using a line graph.

e) Find an example of a misleading graph. Check magazines and newspapers.

Exercise 2: Deceptive Graphs (continued)

3. Look at the graph below.



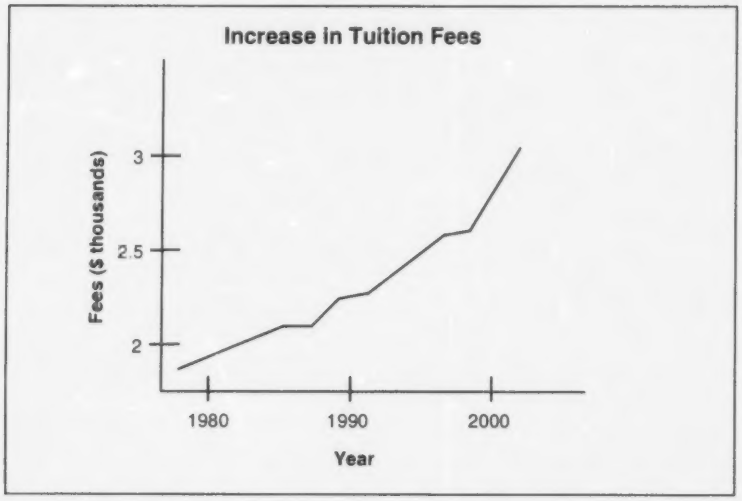
a) This graph shows the percent increase in productivity. Is the company improving or decreasing in productivity?

b) The company produced 2000 units in June. Calculate the number of units produced in July, August, September, and October, assuming that the percentage increases are from one month to the next.

c) Draw a graph showing the percentage changes more truthfully.

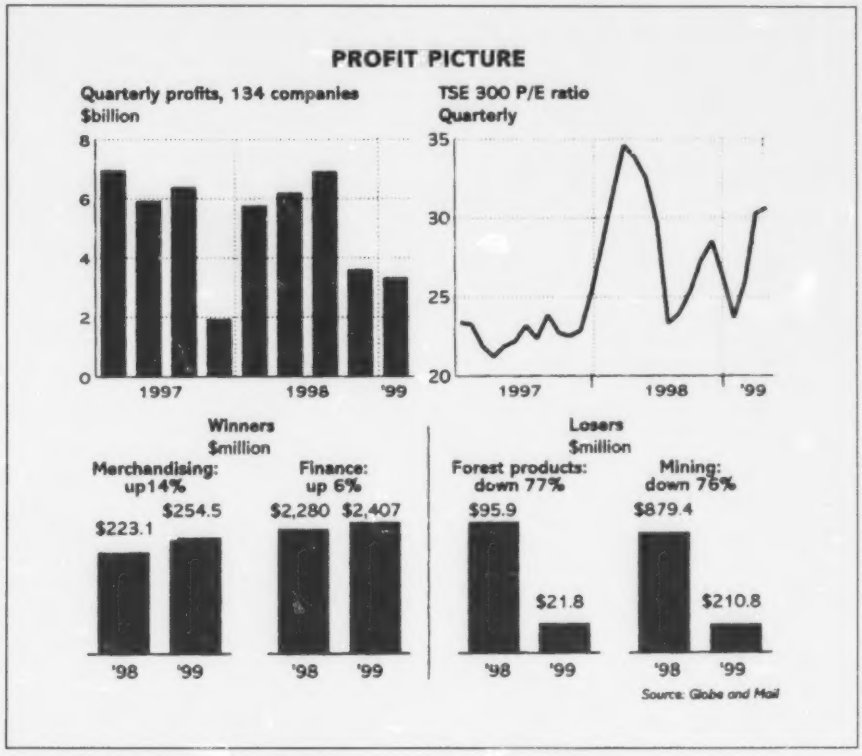
Exercise 2: Deceptive Graphs (continued)

4. a) What impression is given by the graphs below?
   b) How has this impression been created?



Exercise 3: Interpreting Graphs: Stacked and Side-by-Side Graphs

1. Using the graph entitled Profit Picture, answer the following questions:
   a) Give the amount by which profits from financial enterprises increased from 1998 to 1999.
   b) Give the amount by which profits from forest products have decreased from 1998 to 1999.
   c) In which quarters did the profits from "134 companies" reach more than $6 billion?



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Exercise 3: Interpreting Graphs: Stacked and Side-by-Side Graphs (continued)

2. Using the graphs entitled Recent Interest Rate Developments, answer the following questions:
   a) When was the interest rate for Canadian short-term bonds greater than the interest rate for U.S. short-term bonds?
   b) What is the interest rate for U.S. long-term bonds at the end of March 1999?
   c) What is the (approximate) date at which the interest rate for Canadian long-term bonds was lowest?
   d) At which times was the Canadian dollar trading for more than 68¢ U.S.?

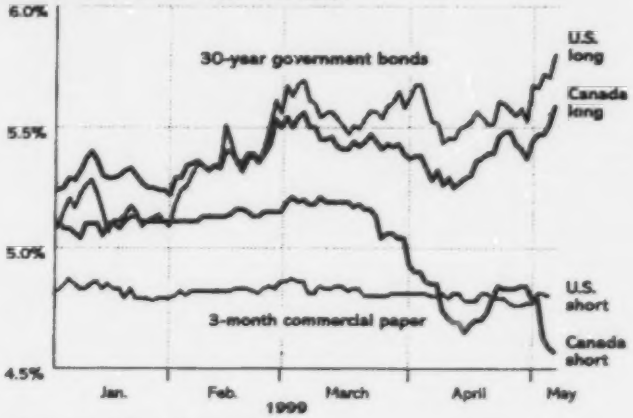
ON THE GRID

Alexandra Eadie

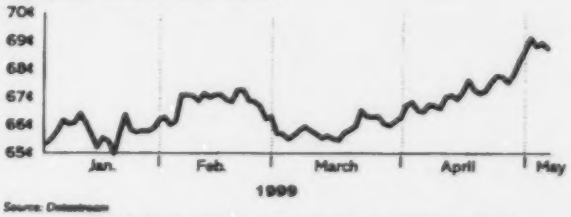
RECENT INTEREST RATE DEVELOPMENTS

Long-bond rates are up significantly in 1999 in the United States and have climbed higher than than the comparable Canadian rates. Short-term interest rates in the U.S. have hardly moved this year in contrast to the Canadian decline of more than half a percentage point. The chart shows that the steep decline in short-term Canadian rates began about the same time as the Canadian dollar started to gain in value.

Canadian and U.S. interest rates (daily)



Canadian dollar (U.S. cents, daily)



Source: Datastream

Alexandra Eadie is a statistician at Report on Business (eadie@globeandmail.ca)

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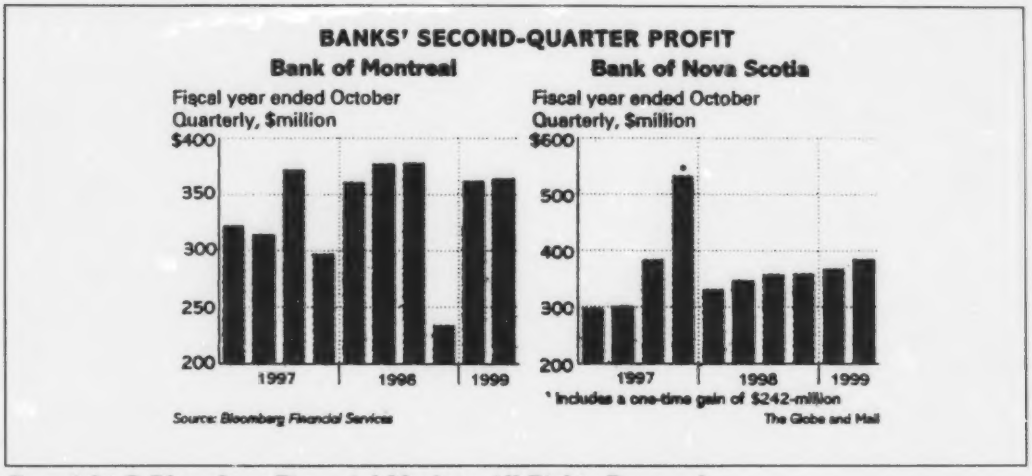
Exercise 3: Interpreting Graphs: Stacked and Side-by-Side Graphs (continued)

3. The graph below compares the quarterly profits ($ millions) for the Bank of Montreal and the Bank of Nova Scotia from 1997 to the second quarter of 1999.



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a) Use the graph to find the following information.

i) Estimate the profits for each bank for each quarter and enter the numbers in a spreadsheet.

ii) Make a side-by-side bar graph to illustrate the situation.

iii) In how many quarters does the Bank of Montreal have higher profits?

iv) In how many quarters does the Bank of Nova Scotia have higher profits?

b) Comment on the effect of the scales chosen for the original graph and for your graph.

Exercise 4: Glyphs

1. Write out your name in hieroglyphics. Remember that when writing in hieroglyphics, vowels are usually omitted and for names containing the letter "l," the Egyptians used "r."

Sign								
Translation	A	I	Y	A	P	W	B	F
Object Depicted	Egyptian Vulture	Flowering Reed	Flowering Reeds	Forearm	Stool	Quail or Chick	Foot	Horned Viper
Sound	A As in Fat	Y	Y As in Hebrew "YODH"	AH As in Father gutural	P	W	B	F

Sign								
Translation	M	N	R	H	H	CH	CH	S
Object Depicted	Owl	Water	Mouth	Reed Sheltering Field	Wick or Twisted Flax	Placenta	Animal's Belly with Teats	Bolt Folded Cloth
Sound	M	N	R	H As in English	H Emphatic	CH As in Scotch Loch	CH As in German ich	S

Sign								
Translation	SH	Q	K	G	T	CH	D	J
Object Depicted	Pool	Hillslope	Basket with Handle	Stand for Jar	Boat	Tethering Rope	Hand	Snake
Sound	SH	Q As in Queen	K	G Hard	T	Ch As in Chest	D	Emphatic

Exercise 4: Glyphs (continued)

2. Four-Dimensional Glyphs

On the next page you will find the glyphs that are circles with as many as four "rays" and that represent data on 20 hospital patients. The length of each ray is proportional to the value of a medical variable, and its position on the circle determines which variable.

The rays represent injury severities to four body subsystems. They are the central nervous system (N), the circulatory system (C), the respiratory system (R), and the renal or kidney system (K). The longer the ray, the more severe the problem. A shorter ray indicates a moderate problem and the absence of a ray indicates a negligible problem.

a) These data have _____ dimensions.

b) Of the survivors, who is "better off,"

— Patient 1 or 7? _____ Why?

— Patient 6 or 9? _____ Why?

c) Which of the non-survivors appeared more likely to survive at the time these glyphs were made:

— 11 or 15?_____ Why?

— 18 or 20?_____ Why?

d) Which patient appears, from his glyph, to be in better condition:

— survivor 5 or non-survivor 15? _____

— survivor 7 or non-survivor 20? _____

e) Which of the following statements seems most appropriate?

i) The chosen indices have little to do with possible live/die status.

ii) The chosen indices accurately describe possible live/die status.

iii) The chosen indices are helpful in describing possible live/die status.

iv) If any one ray in our glyph is long, the patient will probably die.

Question 2: Adapted from Sacco, William, et al. *Glyphs: Getting the Picture*. Copyright © 1987. Used by permission of Everyday Learning Corporation.

Exercise 4: Glyphs (continued)

Medical Glyphs

Survivors		Non-Survivors	
Patient No.	**Glyph**	**Patient No.**	**Glyph**
1		11	
2		12	
3		13	
4		14	
5		15	
6		16	
7		17	
8		18	
9		19	
10		20	

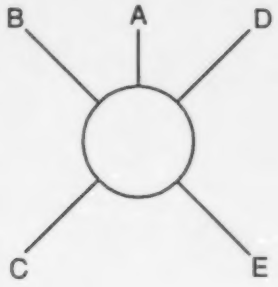
Key

C: Circulatory System
R: Respiratory System
K: Kidney System
N: Central Nervous System

Exercise 4: Glyphs (continued)

3. Multivariate Data

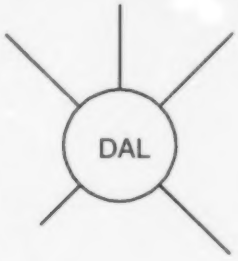
We will now use circle and ray glyphs to examine much different data: the offensive attributes to 10 new football league clubs. We depict the characteristics and effectiveness of these offenses. The data have been obtained from the Unofficial 1998 NFL Statistics Guide.

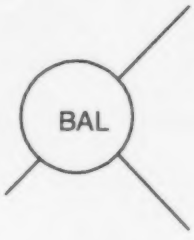


Ray A: represents the number of punts: the longer the ray, the fewer the punts
Ray B: represents the average number of yards per rush (run) attempts
Ray C: represents the number of touchdowns scored on running plays
Ray D: represents the number of completed passes
Ray E: represents the number of touchdowns scored by passing

Note: The longer rays are always preferable to shorter ones.

Here are two sample glyphs:





a) Which team, Dalmeny or Balgonie, scored more points by rushing?
b) Which team, Dalmeny or Balgonie, scored more points by passing?
c) Which team punted more often?
d) The glyph representing Balgonie has no Ray B, that is, the length of Ray B is zero. Does this mean that Balgonie averaged zero yards per attempt?
e) What else might zero length mean?
f) Which team displays a more effective offense?

These glyphs include only the offensive characteristics. While more rays or additional glyphs would give a more complete picture, assume that the other characteristics are equal.

Question 3: Adapted from Sacco, William, et al. *Glyphs: Getting the Picture*. Copyright © 1987. Used by permission of Everyday Learning Corporation.

Exercise 4: Glyphs (continued)

g) Refer to the glyphs below and try to rank the five teams in each division (where team 1 would have won the most games and team 5 the least games).

Group I

Group II

Rank by Sight

Group 1	Group 2
Rank 1 _____	Rank 1 _____
2 _____	2 _____
3 _____	3 _____
4 _____	4 _____
5 _____	5 _____

h) While visual reference to the length of each ray may give a general impression of each team's offensive effectiveness, there is a more precise method to determine the best team. It is a Global Score (GS). The sum of the length of the rays determines the GS.

$$GS = (\text{length of Ray A}) + (\text{length of Ray B}) + (\text{length of Ray C}) + (\text{length of Ray D}) + (\text{length of Ray E})$$

The length of each ray may range from 0 to 10 millimetres.

i) What is the smallest possible GS?

ii) What might this value tell you about a football team?

iii) What is the largest possible GS?

Exercise 4: Glyphs (continued)

i) The following table lists the statistics and the corresponding ray lengths of all 10 listed teams. Rank each group of five teams by computing a GS and list the teams in order within each group.

Team	Punts	Top Ray	Yds/ Run	L. Top Ray	% Pass Comp.	R. Top Ray	T.D.s Running	L. Bot Ray	T.D.s Passing	R. Bot Ray
A	75	8.1	3.5	1.5	62.5	10.0	25	8.4	24	7.4
B	101	0.8	3.3	0.0	56.9	7.2	12	1.6	18	4.8
C	68	10.0	4.6	10.0	55.3	6.5	25	8.9	26	8.3
D	91	3.6	4.2	6.9	53.5	5.6	23	7.4	17	4.3
E	73	8.6	4.2	6.9	51.5	4.6	23	7.4	16	3.9
Z	69	9.7	4.5	9.2	60.0	8.8	28	10.0	16	3.9
Y	93	3.1	4.0	5.4	52.3	5.0	17	4.2	16	3.9
X	98	1.7	3.8	3.8	48.2	3.0	11	1.1	14	3.0
W	81	6.4	4.6	10.0	50.4	4.0	24	7.9	12	2.2
V	76	7.8	4.1	6.2	57.1	7.3	15	3.2	29	9.6
Overall NFL Low	68		3.3		42.2		9		7	
Overall NFL High	104		4.6		62.5		28		30	

i) Calculate the GS for each team and rank the five teams in each group.

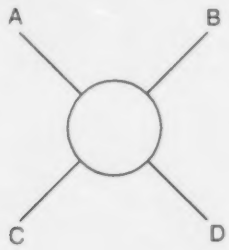
ii) How accurate was your visual ranking from question 2 versus the GS ranking?

iii) If, after some research, you found that the passing percentage (D) was twice as important as any other indicator, how would you rewrite the GS formula?

Exercise 4: Glyphs (continued)

4. International Characteristics

Circle-and-ray glyphs can be used to represent characteristics of countries as follows:



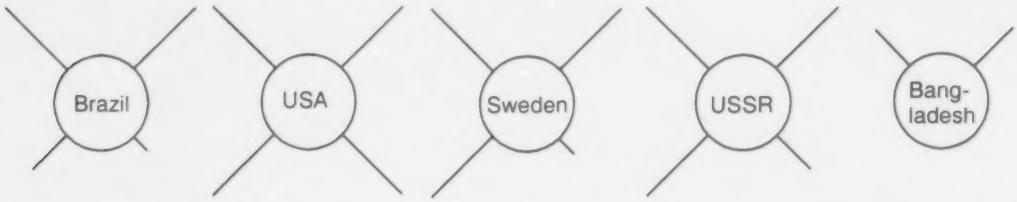
Ray A: literacy rate: the longer the ray, the greater the proportion of literate people

Ray B: life expectancy at birth: the longer the ray, the greater the life expectancy

Ray C: population per square mile: the longer the ray, the less dense the population

Ray D: gross national product (GNP): the total value of the national output of goods and services: the longer the ray, the greater the GNP

These factors are frequently associated with the standard of living. Therefore, the following glyphs can represent standards of living in the five countries.



a) Which country has the lowest life expectancy?

b) Which country has the highest GNP?

c) Rank the five countries according to your conception of "standard of living":

i) _____ (highest)

ii) _____

iii) _____

iv) _____

v) _____

d) What criteria did you use to make your rankings?

e) Compare your answer with the answers of three other students. Did you all use the same criteria? If not, what did other students use that you did not? Would you consider changing your ranking based on this new information? Why or why not?

f) The glyph representing Bangladesh lacks Ray C. Does this mean that the country has no population?

Question 4: Adapted from Sacco, William, et al. *Glyphs: Getting the Picture*. Copyright © 1987. Used by permission of Everyday Learning Corporation.

Exercise 4: Glyphs (continued)

g) The longer Ray C, the less population per square mile. What can a zero length mean?

h) Assigning lengths to rays requires formulas. For the following question, use the information below.

Ray A: literacy rate x 10 = number of millimetres

Ray B: life expectancy ÷ 8 = number of millimetres

Ray C: 10 – (population/sq. mile ÷ 50) = number of millimetres

Ray D: GNP ÷ 200 billion = number of millimetres

Data for Mexico

Ray A: Literate population	0.84
Ray B: Life expectancy	60
Ray C: Population/square mile	86
Ray D: GNP	80.3

i) Draw the glyph of Mexico.

ii) Are there other important factors that should be added to the glyphs in order to portray a better representation of the standard of living in a country? What are they? State a rationale as to why these factors should be included.

i) Construct glyphs to represent the standard of living in the following countries. Use the following information.

Ray A: 10 – (infant mortality rate ÷ 50) = number of millimetres

Ray B: life expectancy ÷ 8 = number of millimetres

Ray C: 10 – (population/sq. mile ÷ 50) = number of millimetres

Ray D: GNP ÷ 2500 = number of millimetres

Country	Life Expectancy	Infant Mortality Rate	Population/ Square Mile	GNP
Burkina Faso	47	103	103	$230
Canada	78	6	8	$19 380
Brazil	67	48	49	$3 640
Bangladesh	58	77	2432	$240
Japan	80	4	867	$39 640
France	78	5	276	$24 990
Russia	65	18	23	$2 240
Ethiopia	47	120	152	$100

Exercise 4: Glyphs (continued)

i) Rank the countries according to your conception of standard of living.

a) _____ e) _____

b) _____ f) _____

c) _____ g) _____

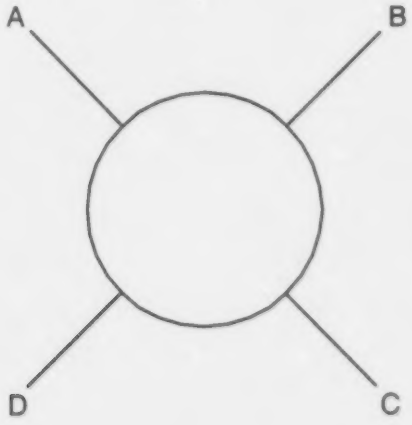
d) _____ h) _____

ii) What criteria did you use to make your rankings?

iii) Compare your rankings with those of two other students. Did you all use the same criteria? If not, what did the other students use that you did not? Would you consider changing your rankings based on this new information?

iv) The ray representing Bangladesh lacks Ray D. Does this mean that the country has no population? Explain.

v) Are there any other important factors that should be added to glyphs in order to represent better the standard of living of a country? What are these factors? State a rationale as to why they were included.

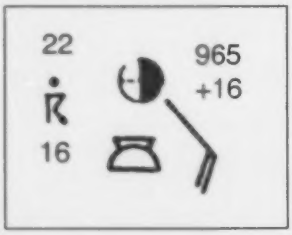


Exercise 4: Glyphs (continued)

5. Weather Plots

Using the attached chart, answer the following questions:

a) Winnipeg



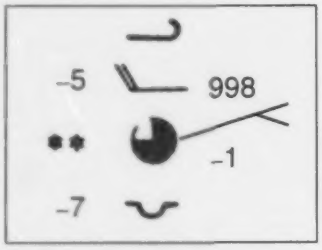
i) What is the temperature?

ii) What is the present state of the weather?

iii) What is the wind speed and direction?

iv) Describe the clouds.

b) Brandon



i) What is the temperature?

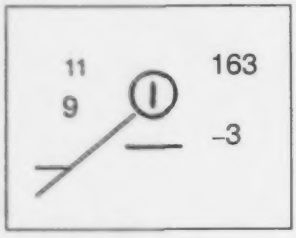
ii) What is the present state of the weather?

iii) What is the wind speed and direction?

iv) Describe the clouds.

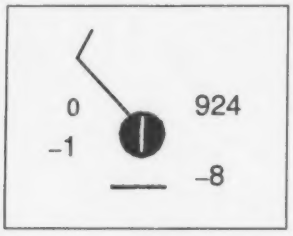
c) Describe the weather in each of the following places.

i) San Francisco

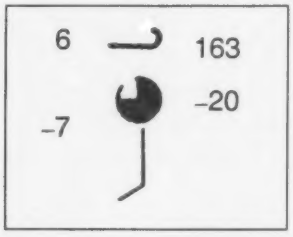


Exercise 4: Glyphs (continued)

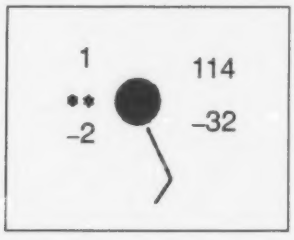
ii) Yellowknife



iii) Churchill



iv) The Pas



d) Make weather plots for three consecutive days in your area. Show the sky cover, the wind direction and speed, the present state of the weather, and the cloud types.

Simplified Station Model

Explanations of the various symbols and codes of a weather glyph example (see below):

Pressure: When sea-level pressure is plotted, the 1000s digit, the 100s digit, and the decimal point are omitted. A reported pressure of 1012.3 mb is plotted as 123, and 996.7 as 967.

Temperatures: Plotted in whole degrees Celsius.

Dew point: Plotted in whole degrees Celsius.

Wind: The point on the station circle from which wind shaft emerges shows the direction from which the wind is blowing. Wind direction on station model is northwest.

Wind speed: Wind speed is determined by the number of feathers on the shaft (see Chart 1).

Sky cover: Chart 2 indicates how to determine the amount of sky covered by clouds.

Tendency (pressure change): Data immediately to the right of the station circle show barometer behaviour three hours before the observation. If the pressure is higher than it was three hours ago, a plus sign is used; if lower, a minus sign. The number represents the pressure differences in tenths of a millibar, with the decimal point omitted. The example shows that the barometric pressure is 1.6 mb higher than it was three hours ago.

Clouds: Low clouds are plotted immediately underneath the station circle; middle clouds immediately above; and high clouds immediately above the middle clouds. If no middle clouds are reported, high clouds are plotted immediately above the station circle. Some of the more commonly observed clouds appear in Chart 3.

Weather: Present state of the weather is plotted to the immediate left of station circle. Symbols appearing most frequently on maps are shown on Chart 4.

Weather Glyph Example

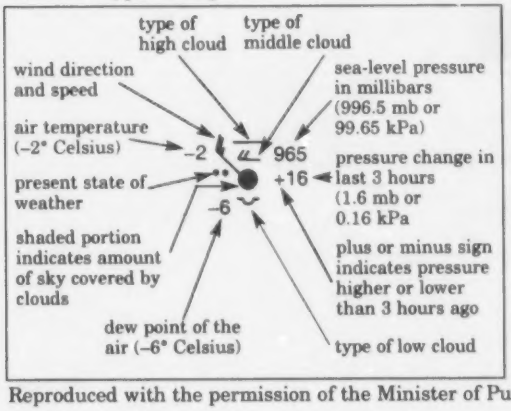

type of high cloud
type of middle cloud
wind direction and speed
sea-level pressure in millibars (996.5 mb or 99.65 kPa)
air temperature (–2° Celsius)
–2
965
pressure change in last 3 hours (1.6 mb or 0.16 kPa
present state of weather
+16
–6
shaded portion indicates amount of sky covered by clouds
plus or minus sign indicates pressure higher or lower than 3 hours ago
dew point of the air (–6° Celsius)
type of low cloud

Chart 1

SPEED	SYMBOL	SPEED	SYMBOL
(2-4 km/h)		(70-78 km/h)	
(5-13 km/h)		(79-87 km/h)	
(14-22 km/h)		(88-96 km/h)	
(23-31 km/h)		(97-105 km/h)	
(32-41 km/h)		(106-115 km/h)	
(42-50 km/h)		(181-189 km/h)	
(51-59 km/h)		(190-198 km/h)	
(60-69 km/h)		(199-207 km/h)	

Note: A circle around the station location indicates calm wind.

Chart 2 **Sky Cover**

0		Absolutely no clouds	5		5/8
1		1/8 or less	6		6/8
2		2/8	7		7/8
3		3/8	8		8/8 (overcast)
4		4/8	9		sky obscured

Chart 3 **Major Cloud Types**

HIGH CLOUDS base above 6000 m	Thin Cirrus	Thick Cirrus	Tufted Cirrus	Cirro-stratus	Cirro-cumulus	
MIDDLE CLOUDS base between 2000 m-6000 m	Thin Alto-stratus	Thick Alto-stratus/ Nimbo-stratus	Alto-cumulus	Alto-cumulus in small patches	Alto-cumulus in bands	Alto-cumulus with Alto-stratus
LOW CLOUDS base below 2000 m	Fair weather cumulus	Swelling cumulus	Cumulo-nimbus	Strato-cumulus	Cumulus flattened to Strato-cumulus	Stratus Fractus and/or Cumulus Fractus

Chart 4 **Present State of the Weather**

N.B. A bracket to the right of the symbol means: occurred during last hour, but not at time of observation.

,	intermittent light drizzle	:	intermittent moderate snow	(≡)	fog in sight
,,	steady light drizzle	::	steady light snow	≡	fog
.	intermittent light rain	⁝	light snow shower	⌈	heavy thunderstorm
⁘	steady heavy rain	=	mist	⁎	light or moderate thunderstorm with rain
⁝	light rain shower	<	lightning	⌐	smoke

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Exercise 4: Glyphs (continued)

6. Projects

Choose one of the following projects on glyphs that are familiar to you. Below are two suggested topics.

Choice A

Refer to a consumer buying guide to research a hypothetical purchase. Using the information from the guide — both written, glyph, and graph — choose what you think is the best buy. Justify your choice based on the data provided, including cost, features, repairs, and rating details. Provide a copy of the relevant data used in the guide. Present your findings in writing and/or in an oral presentation (teacher's decision).

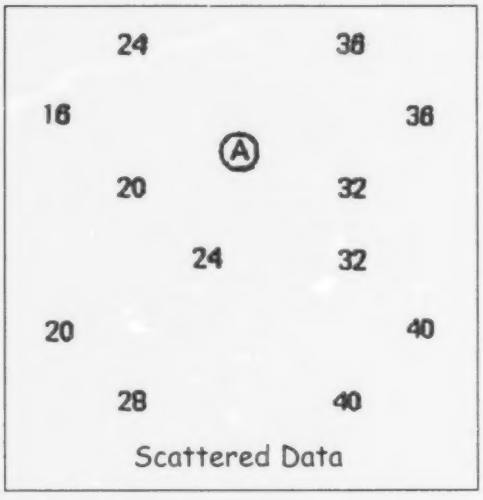
Choice B

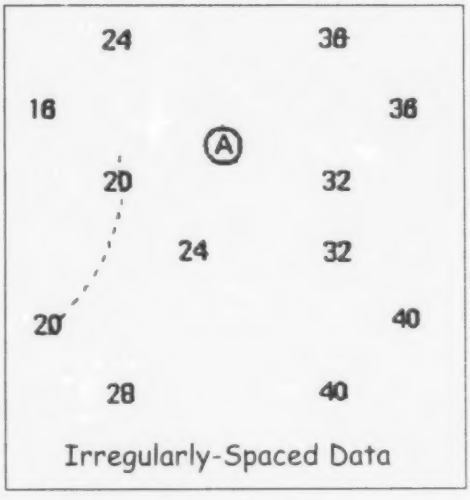
Find examples of glyphs found in your environment. Sketch and/or photograph these glyphs and explain your interpretation of the glyphs and why they are used, for example, the "hand" in red for "Don't Walk" on traffic lights.

Exercise 5: How to Construct Simple Contour Maps

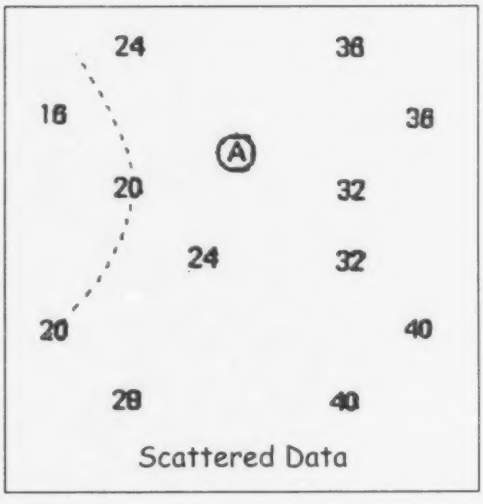
1. Construct a Simple Contour Map

   Step 1: Start with a plot of scattered data points. To determine the value at point A, you must draw some contour lines first. Basically, a contour line is a line that connects locations that have the same data value. To draw the contour line for 20 degrees, draw a line connecting all of the points that are 20 degrees.





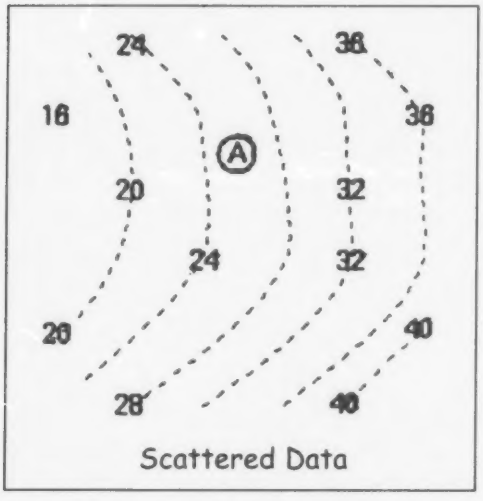
Step 2: Extend the line (interpolation) between two points where one value is higher and one is lower.



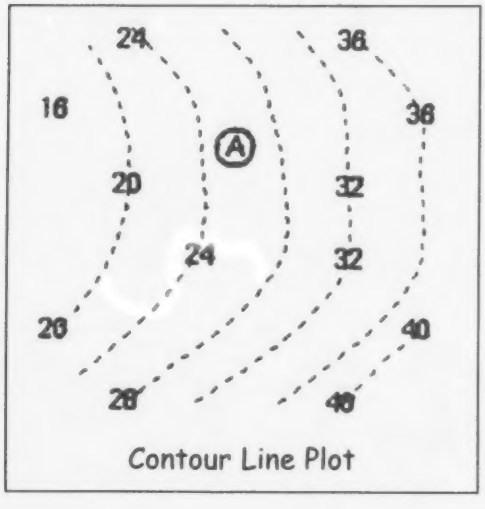
Exercise 5: How to Construct Simple Contour Maps (continued)

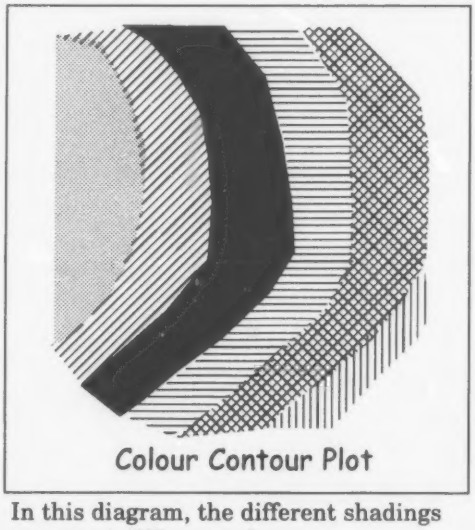
Step 3: Now that the contour line for 20 degrees is complete, any point to the left of the line is colder than 20 degrees, and any point to the right of the line is warmer than 20 degrees.

Step 4: This procedure is repeated until you have all of the lines you need. Usually contour lines are drawn at fixed intervals such as every 2 or 4 degrees.



Step 5: You can either label the lines, or fill the regions between the lines with colour.

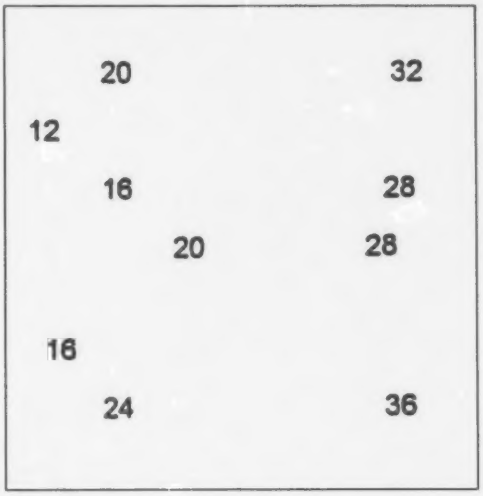




In this diagram, the different shadings represent different colours.

Exercise 5: How to Construct Simple Contour Maps (continued)

2. Construct a contour map using the following data points. Use intervals of 4 degrees between each contour line (i.e., 12, 16, 20, 24, 28, 32, 36 degrees).



Once the contour lines are connected, label them.

Exercise 5: How to Construct Simple Contour Maps (continued)

3. Draw a contour map for solar radiation mega-joules per square metre (MJ/m^2).

Exercise 5: How to Construct Simple Contour Maps (continued)

4. Draw a contour map for the growing season in days.

Exercise 5: How to Construct Simple Contour Maps (continued)

5. Draw a contour map for growing degree days above 6°C.

Exercise 5: How to Construct Simple Contour Maps (continued)

6. Construct a contour map using the following temperatures (°C). Use a pencil to draw contour lines. Use intervals of 2 or 3 degrees.

Exercise 5: How to Construct Simple Contour Maps (continued)

7. Create a contour map for the high temperatures across Canada for a day in the past week.

You may need to consult the following web site:
Environment Canada, Canadian Meteorological Centre
http://weather.ec.gc.ca

Unit F
Data Management and Analysis
Solutions

Exercise 1: Time Series Plots — Solutions

1. a) 1051; 245

   b) 68 (405/6)

   c) 2; 8

   d) About 1978; about 1.4 (510/365)

   e) About 580

   f) About 1962; Introduction of diet drinks and aluminum cans

   g) There really isn't an "average person." This phrase is used to represent the mean amount consumed per person; it is computed by dividing the total number of 12-ounce cans consumed by the total number of people in the United States.

   h) From 1945 to 1961, consumption rose gradually from 90 to 130 twelve ounce servings per year per person. In 1962, it rose to about 400 12-ounce servings or it almost tripled. Possible ideas:
   - diet drinks were introduced
   - aluminum cans became available
   - the economy improved and people spent more on luxuries such as soft drinks
   - post-war, baby-boom kids were of teenage years

   Between 1975 and 1980, there were continual increases, then it took about 10 years to realize significant growth. By the late 1980s, consumption seems to have levelled off possibly due to the increasing popularity of fruit juice drinks, and natural and flavoured bottled water, and ice-tea drinks.

2. a) 50%;

   No;

   The percentage is less than 50%.

   b) 21 years, 11 months

3. a) The median age at first marriage for both men and women declined overall from 1900 to 1950, including a marked decline between 1940 and 1950. Since 1950, the median age has been increasing steadily.

   b) The decade from 1940-1950. Answers will vary. Possible conclusions: The war might have caused some uncertainty about getting married; plus, men were overseas and unavailable to marry until their return. After the war, fewer soldiers were overseas, making it easier for couples to get married at younger ages.

Exercise 1: Time Series Plots — Solutions (continued)

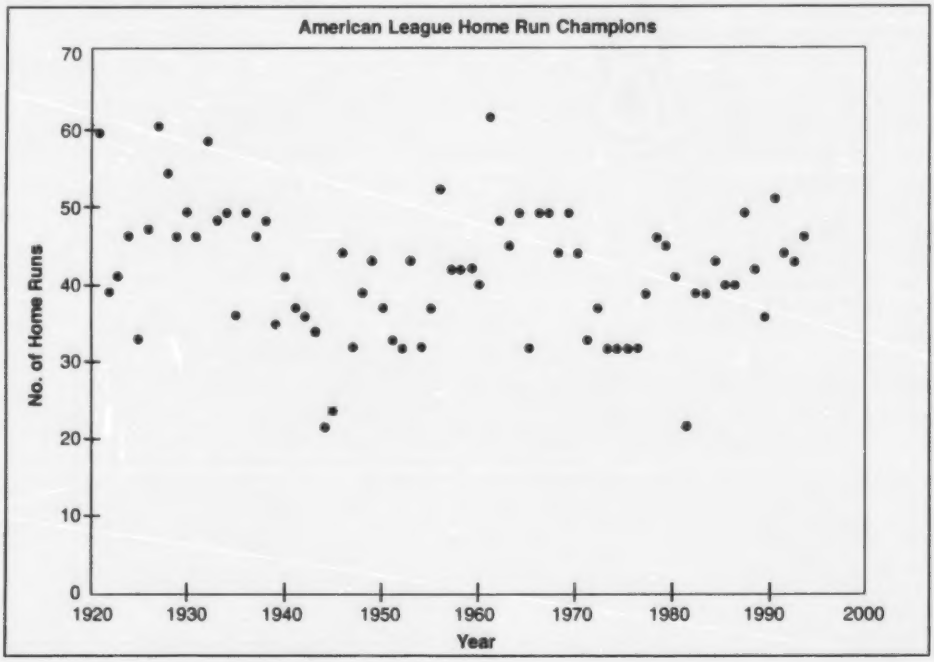
c) i)

ii) There is a steady decrease in the difference of ages except for the period of 1970-1980.

iii) About 27 males and 25 females; about 2 years

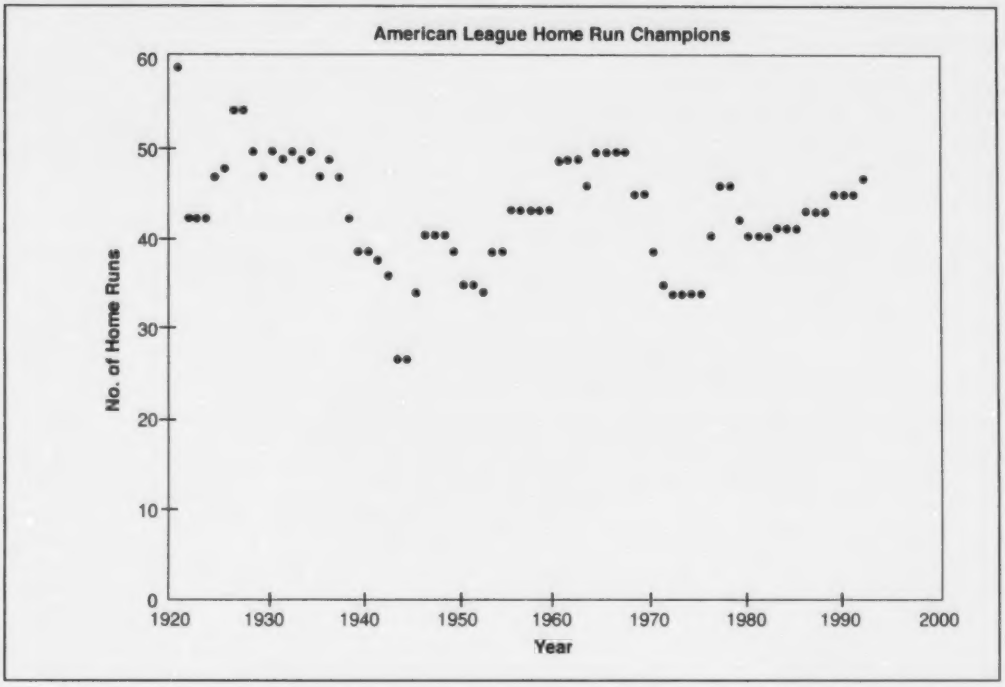
Answers will vary. Sample: The age difference in 2000 should be within the range of 2.3 to 2.0 as the difference in the median ages is consistently decreasing over the years.

4. a) Scatterplot and original data



Exercise 1: Time Series Plots — Solutions (continued)

Scatterplot of the smoothed data.



Exercise 1: Time Series Plots — Solutions (continued)

c)

Year	Home Runs	Smoothed Values	Year	Home Runs	Smoothed Values
1921	59	59	1958	42	42
1922	39	41	1959	42	42
1923	41	41	1960	40	42
1924	46	41	1961	61	48
1925	33	46	1962	48	48
1926	47	47	1963	45	48
1927	60	54	1964	49	45
1928	54	54	1965	32	49
1929	46	49	1966	49	49
1930	49	46	1967	49	49
1931	46	49	1968	44	49
1932	58	48	1969	49	44
1933	48	49	1970	44	44
1934	49	48	1971	33	37
1935	36	49	1972	37	33
1936	49	46	1973	32	32
1937	46	48	1974	32	32
1938	48	46	1975	32	32
1939	35	41	1976	32	32
1940	41	37	1977	39	39
1941	37	37	1978	46	45
1942	36	36	1979	45	45
1943	34	34	1980	41	41
1944	22	24	1981	22	39
1945	24	24	1982	39	39
1946	44	32	1983	39	39
1947	32	39	1984	43	40
1948	39	39	1985	40	40
1949	43	39	1986	40	40
1950	37	37	1987	49	42
1951	33	33	1988	42	42
1952	32	33	1989	36	42
1953	43	32	1990	51	44
1954	32	37	1991	44	44
1955	37	37	1992	43	44
1956	52	42	1993	46	45
1957	42	42			

d) World War II

e) Probably

Table: From *Exploring Data* by James M. Landwehr and Ann E. Watkins. ©1996 by AT&T Corp. Published by Dale Seymour Publications, an imprint of Pearson Learning. Used by permission.

Exercise 1: Time Series Plots — Solutions (continued)

f) 1926 — no; 1931 — no; 1959 — no; 1969 — no; 1971 — possibly, as this might make pitchers less cautious about hitting batters, thereby giving an advantage to pitchers and decreasing home runs

g) Original data — yes; smoothed data — no

h) Use the weighted average of 2/3 of 1921 value plus 1/3 of the 1992 value.

i) About 50

j) Answers will vary. Some possible points:

original plot

- hard to evaluate overall trends
- easy to see outstanding players

smoothed plot

- easier to see overall trends
- easier to attribute historical influences (wars)
- sinusoidal nature of data is more apparent

5. a) 1984

b) 6.42 feet; 7 feet 10 inches

c) 1940 and 1944; World War II

d) West Germany and East Germany were two separate countries that merged into one country, Germany, between 1988 and 1992.

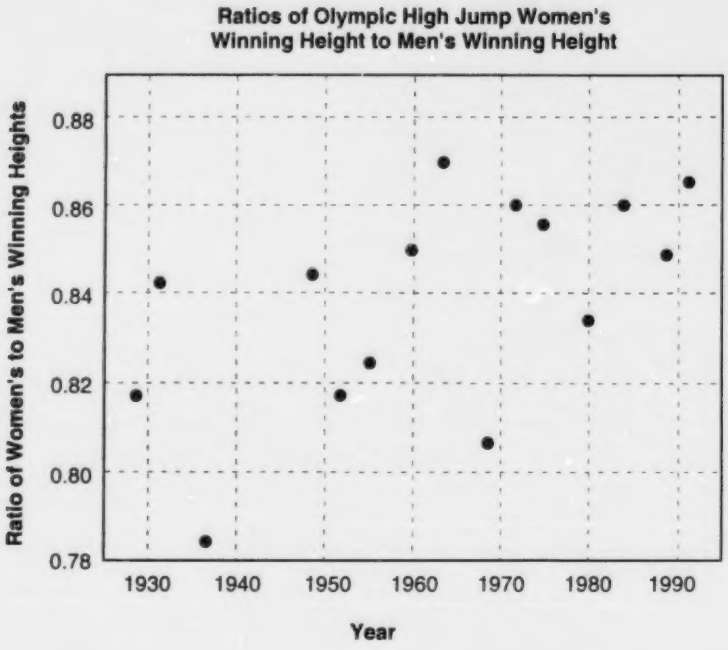
e)

Year	Ratio
1976	0.856
1980	0.836
1984	0.860
1988	0.854
1992	0.864

Question 5 Solution: From *Exploring Data* by James M. Landwehr and Ann E. Watkins. ©1996 by AT&T Corp. Published by Dale Seymour Publications, an imprint of Pearson Learning. Used by permission.

Exercise 1: Time Series Plots — Solutions (continued)

f)

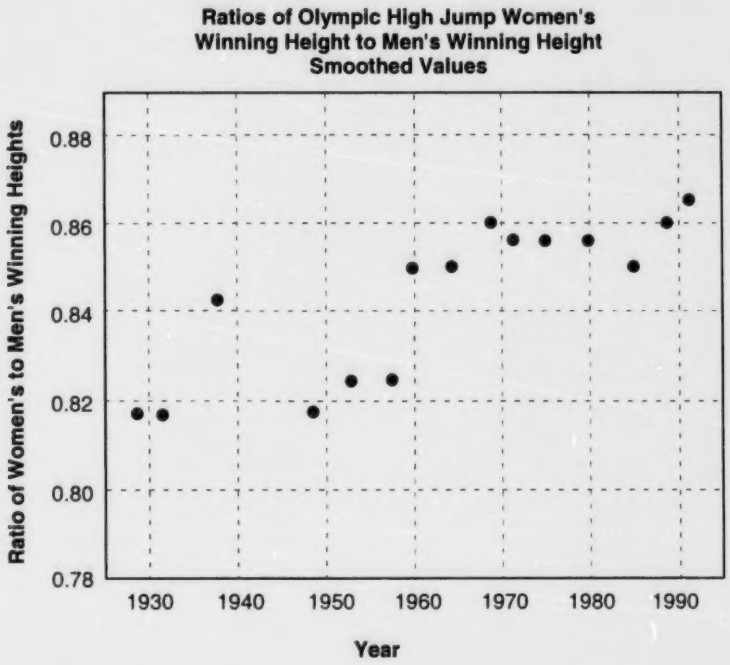


g) Table of ratios and smoothed values

Year	Ratio	Smoothed Values
1928	0.817	0.817
1932	0.842	0.817
1936	0.787	0.842
1948	0.846	0.817
1952	0.817	0.829
1956	0.829	0.829
1960	0.856	0.856
1964	0.871	0.856
1968	0.811	0.860
1972	0.860	0.858
1976	0.858	0.858
1980	0.836	0.858
1984	0.860	0.854
1988	0.854	0.860
1992	0.864	0.864

Exercise 1: Time Series Plots — Solutions (continued)

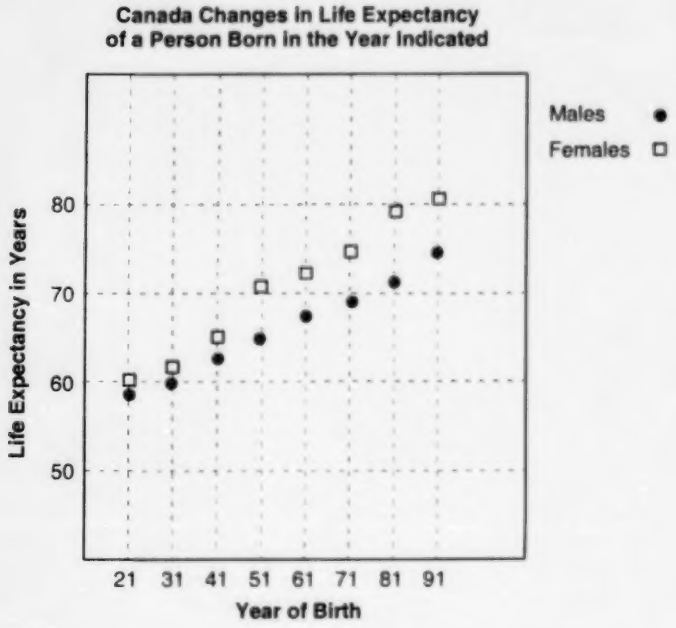
h)



i) The ratio remains relatively steady until 1960, at which point it increases and continues to increase gradually after that. The Olympics did not take place in 1940 and 1944, so the smoothed pattern for 1936 and 1948 may be a little distorted. A value of 1 will not be reached quickly. A line of best fit would give an indication, but since the increase is so gradual and the smoothed values would be used, you may not have much confidence in the result. If the ratio is equal to 1, the women's winning high jump height would be equal to the men's winning high jump height.

Exercise 1: Time Series Plots — Solutions (continued)

6.

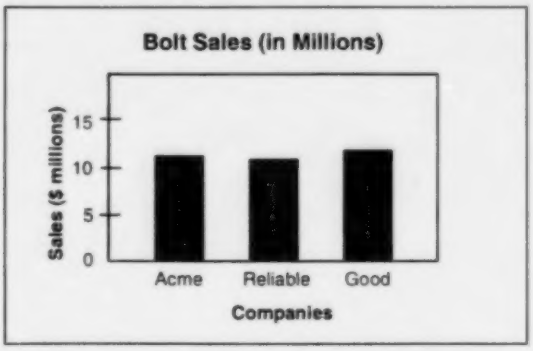


7.

Exercise 2: Deceptive Graphs — Solutions

1. a) That Good Bolt Company sales far exceeds the others. It has been created by the small scale of increments for sales. If it had been in a scale of 5 000 000 increments then the graph would have been more honest.

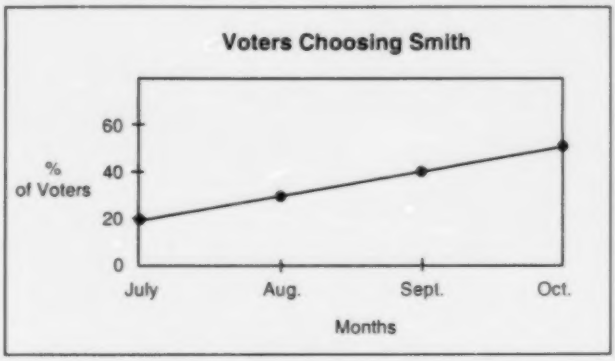
b)



2. a) Smith is gaining.

b) Months should be shown horizontally, percentages should be shown vertically.

c)



d) Students may give answers that are correct but not included here.

Bar

- increase width of certain bars to make one category seem bigger
- do not begin scale at zero
- distort vertical/horizontal scale by increasing it or decreasing it

Line

- reverse of vertical or horizontal labels
- distort vertical scale

Exercise 2: Deceptive Graphs — Solutions (continued)

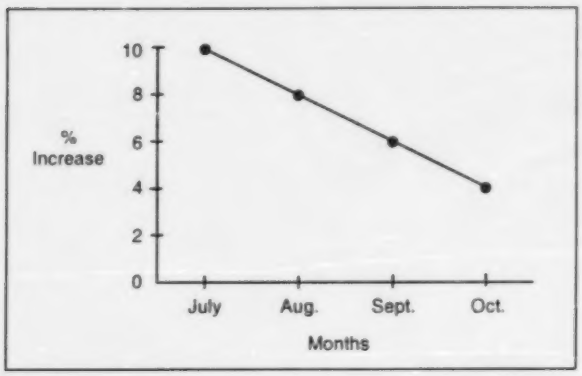
3. a) Although it seems that productivity is improving, the reversed scales make it difficult to see the true situation.

   The basis for the percentage is not shown. Is it from 1 month to the next? Is it from the same month in the previous year? There is no title.

   b)

Month	Production
July	110% of 2000 = 2200
August	108% of 2200 = 2376
September	106% of 2376 = 2518
October	104% of 2519 = 2618

   c)



4. a) Students' tuition has increased more rapidly than professors' salaries.

   b) Money scales are different.

   Time period illustrated is different.

   Vertical scales do not begin at zero.

Exercise 3: Interpreting Graphs — Solutions

1. a) $127 million

   b) $74.1 million

   c) 1997 — 1st and 3rd

   1998 — 2nd and 3rd

2. a) From January 1st until about April 7th and again from about April 20th until May 2nd.

   b) Rate for U.S. long-term bonds at end of March ≈ 5.7%

   c) January 31st

   d) From May 1st on

3. a) i) and ii). See spreadsheet and graph below. Numbers may vary.

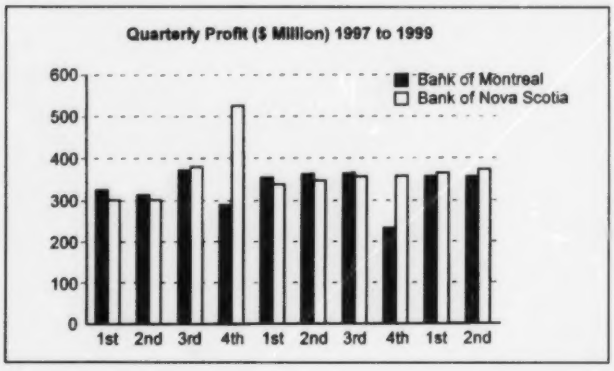
   iii) The Bank of Montreal has higher profits in 5 of the quarters.

   iv) The Bank of Nova Scotia has higher profits in 5 of the quarters.

   b) The original graphs have different vertical scales; Bank of Montreal from $200 million to $400 million and Bank of Nova Scotia from $200 million to $600 million.

   At first glance, it seems that Bank of Montreal has considerably higher profits in almost every quarter. The side-by-side graph has scale from $0 million to $600 million and the profit comparison is seen more easily.

	A	B	C
1	Quarterly Profit $ Million 1997 to 1999	Bank of Montreal	Bank of Nova Scotia
2	1st	320	300
3	2nd	310	300
4	3rd	370	380
5	4th	295	520
6	1st	360	340
7	2nd	375	350
8	3rd	376	360
9	4th	230	360
10	1st	360	370
11	2nd	360	380



Exercise 4: Glyphs — Solutions

1. Answers will vary.

2. a) 4

   b) Patient 1; Patient 9 — only one problem area

   c) Patient 15; Patient 18 — fewer problem areas

   d) Non-survivor 15; Survivor 7 — fewer and less intense problems

   e) iii)

3. a) Dalmeny

   b) Balgonie

   c) Balgonie

   d) No

   e) The ray length is "normalized" so that 0 length matches the team with the fewest yards per run, and 10 mm matches the team with the most yards per run.

   f) Dalmeny

   g) Varied results possible. Best answer — Group I: C A E D B; Group II: Z W V Y X

   h) i) Zero

      ii) This is probably a poor team.

      iii) 50

   i) i)

GS	A	B	C	D	E
	35.4	14.4	43.7	27.8	31.4

Ranking: C A E D B

GS	Z	Y	X	W	V
	41.6	21.6	12.6	30.5	34.1

Ranking: Z V W Y X

   ii) Varied responses possible

   iii) GS = A + B + C + 2D + E

4. a) Sri Lanka

   b) Canada

   c) Possible answer: (i) Canada, (ii) Slovania, (iii, iv) Columbia or Norway (either way), (v) Sri Lanka

   d) If the answer given is the same as (c), then the ranking was done solely on the length of the rays and giving each category of ray equal weight. Materialists may give greater weight to the GNP ray, while others may consider literacy more important.

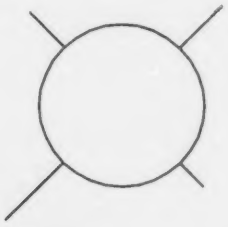
   e) Answers may vary.

   f) No

   g) Extremely large population per square mile

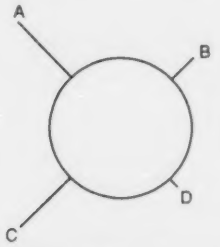
Exercise 4: Glyphs — Solutions (continued)

h) i)

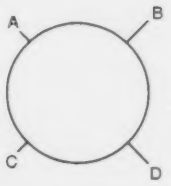


ii) Varied responses possible, e.g., natural resources, tourism

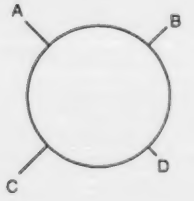
i)



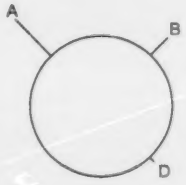
Burkino Faso
A = 103 ÷ 50 x 10 = 20.6 mm
B = 47 ÷ 8 = 5.875 mm
C = 103 ÷ 50 x 10 = 20.6 mm
D = 230 ÷ 2500 = 0.092 mm



Canada
A = 6 ÷ 50 x 10 = 1.2 mm
B = 78 ÷ 8 = 9.75 mm
C = 8 ÷ 50 x 10 = 1.6 mm
D = 19 380 ÷ 2500 = 7.752 mm

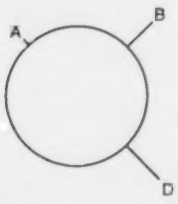


Brazil
A = 48 ÷ 50 x 10 = 9.6 mm
B = 67 ÷ 8 = 8.375 mm
C = 49 ÷ 50 x 10 = 9.8 mm
D = 3640 ÷ 2500 = 1.456 mm

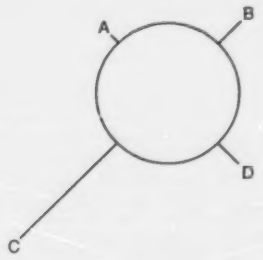


Bangladesh
A = 77 ÷ 50 x 10 = 15.4 mm
B = 58 ÷ 8 = 7.25 mm
C = 2432 ÷ 50 x 10 = 486.4 mm
D = 240 ÷ 2500 = 0.096 mm

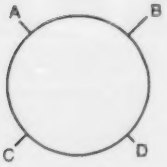
Exercise 4: Glyphs — Solutions (continued)



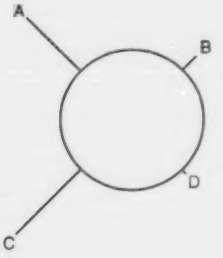
Japan
A = 4 ÷ 50 x 10 = 0.8 mm
B = 80 ÷ 8 = 10 mm
C = 867 ÷ 50 x 10 = 173.4 mm
D = 39 640 ÷ 2500 = 15.856 mm



France
A = 5 ÷ 50 x 10 = 1 mm
B = 78 ÷ 8 = 9.75 mm
C = 276 ÷ 50 x 10 = 55.2 mm
D = 24 990 ÷ 2500 = 9.996 mm



Russia
A = 18 ÷ 50 x 10 = 3.6 mm
B = 65 ÷ 8 = 8.125 mm
C = 23 ÷ 50 x 10 = 4.6 mm
D = 2240 ÷ 2500 = 0.896 mm



Ethiopia
A = 120 ÷ 50 x 10 = 24 mm
B = 47 ÷ 8 = 5.875 mm
C = 152 ÷ 50 x 10 = 30.4 mm
D = 100 ÷ 2500 = 0.04 mm

i) a) Canada
b) France
c) Japan
d) Russia
e) Brazil
f) Bangladesh
g) Burkino Faso
h) Ethiopia

ii) Life Expectancy and Infant Mortality

iii) Answers will vary.

iv) It just means that their GNP is extremely low.

v) Answers will vary.

Exercise 4: Glyphs — Solutions (continued)

5. a) Winnipeg
   i) 22°C
   ii) Light thunderstorm with rain
   iii) Wind from SSE at 32-41 km/hr
   iv) Low cumulo-nimbus clouds and alto-cumulu[illegible] small patches

   b) Brandon
   i) –5°C
   ii) Steady light snow
   iii) WNW at 5-13 km/hr
   iv) Strato-cumulus low clouds; thick alto-stratus and high thin cirrus

   c) i) San Francisco
   Wind from SW at 5-13 km/hr
   Air temperature 11°C
   Low clouds stratus fractus, one-eighth or less cloud cover
   Pressure 1016.3 mb fallen by 3 mb in last 3 hours
   Dew point at 9°C

   ii) Yellowknife
   Wind from NW at 14-22 km/hr
   Air temperature 0°C
   Low clouds stratus fractus, seven-eighths cover
   Pressure 992.4 mb fallen by 8 mb in last 3 hours
   Dew point at –1°C

   iii) Churchill
   Wind from S at 14-22 km/hr
   Air temperature 6°C
   High clouds thin cirrus covering six-eighths of sky
   Pressure 1016.3 mb fallen by 20 mb in last 3 hours
   Dew point at –7°C

   iv) The Pas
   Wind from SSE at 14-22 km/hr
   Steady light snow
   Overcast sky
   Pressure 1011.4 mb fallen by 32 mb in last 3 hours
   Dew point at –2°C

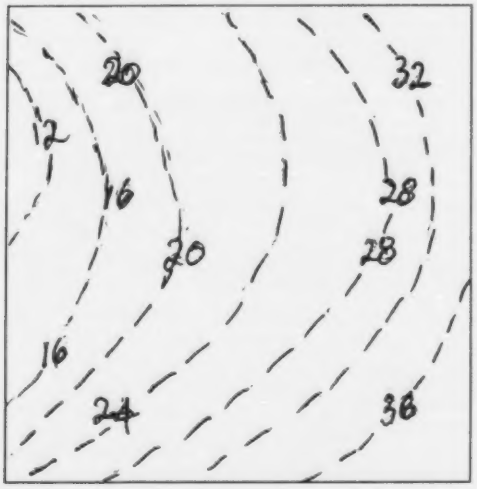
6. Choice B: Based on the student's community, a scale of grades may be developed based on the number of glyphs found, for example, a student living in a large centre.

Grade	No. of Glyphs
A	20
B	17
C	14
D	12
E	<10

Exercise 5: Contour Maps — Solutions

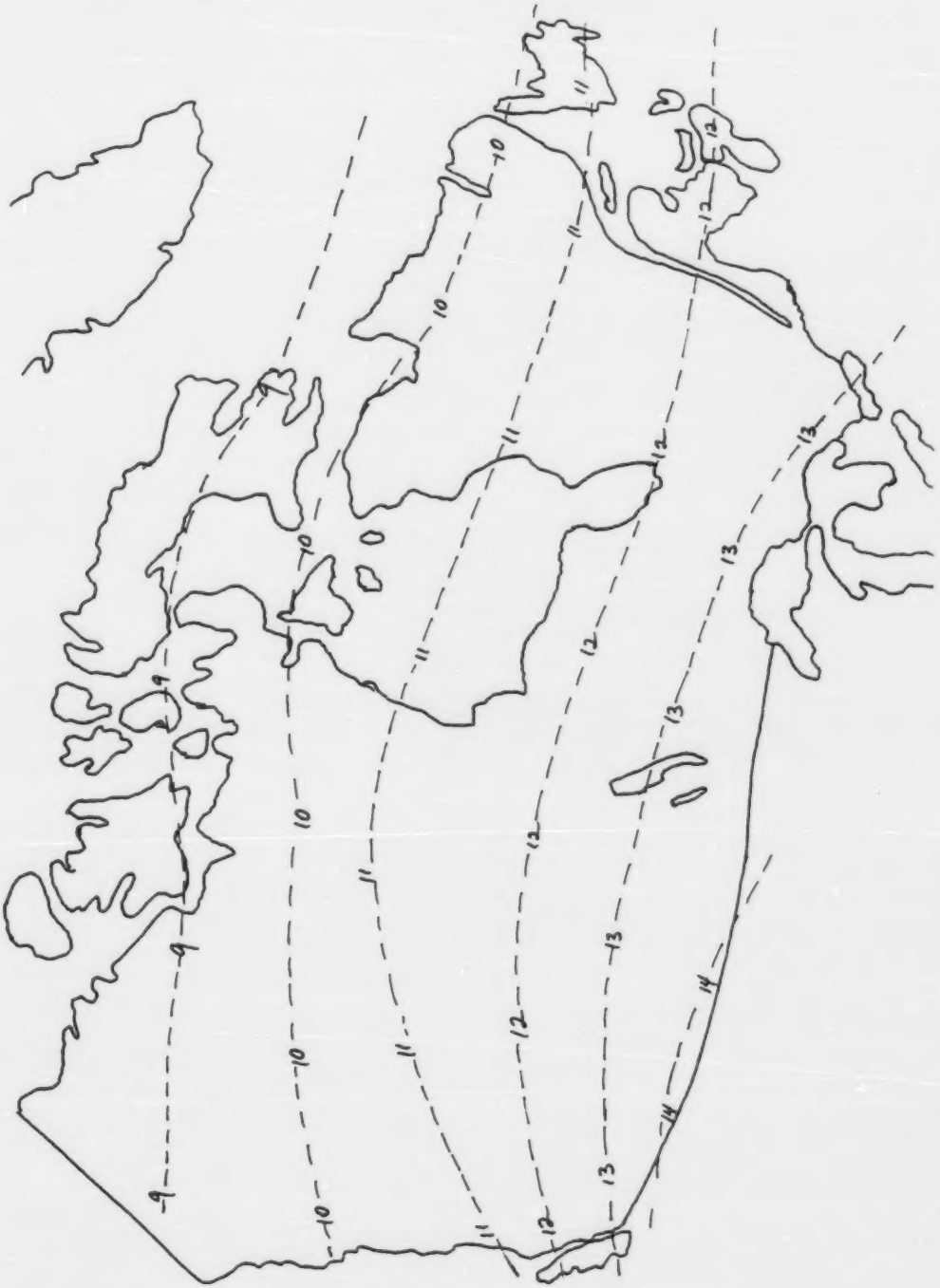
1. Activity — see problems.

2. Answers will vary. Sample solution shown below.



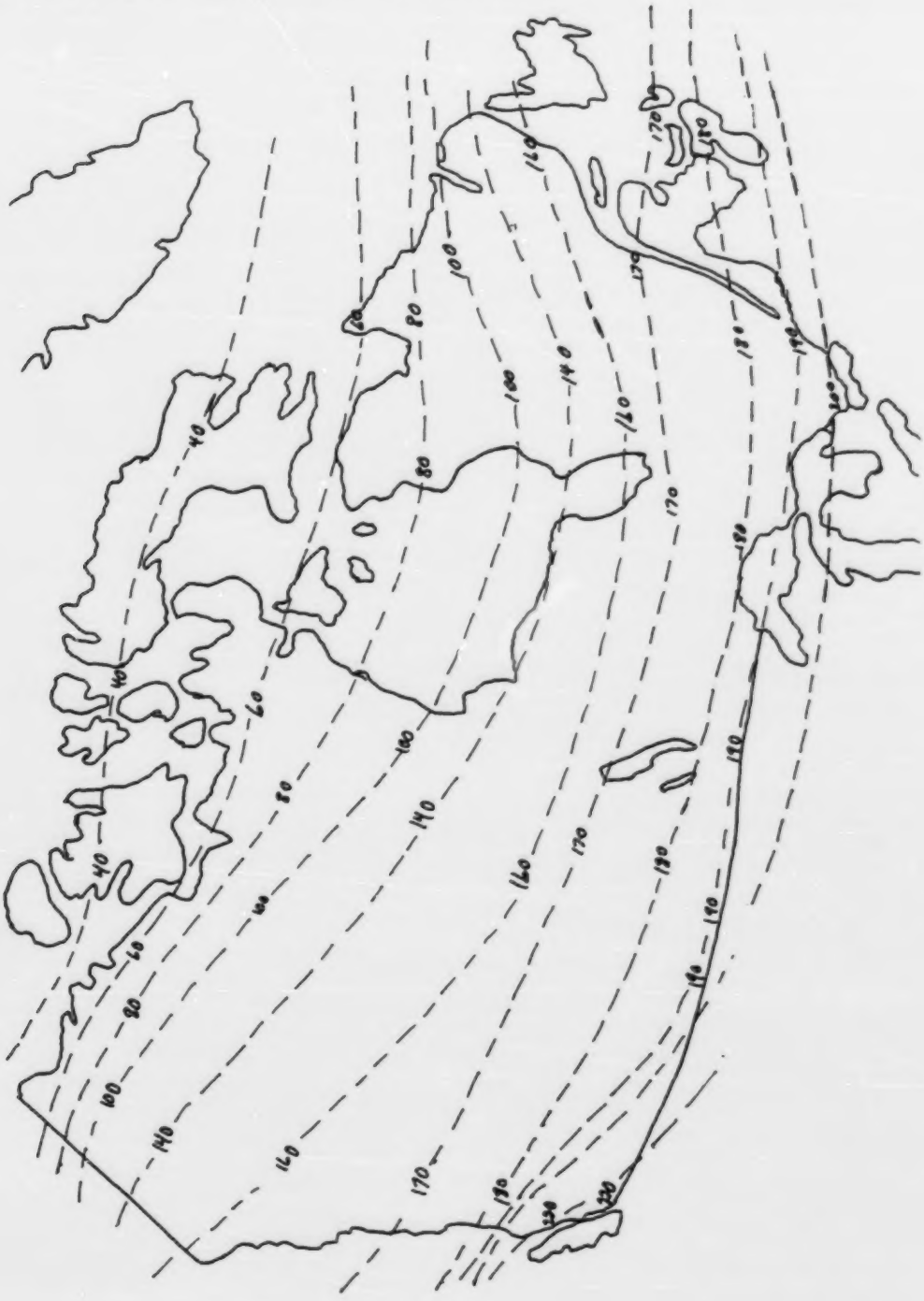
Exercise 5: Contour Maps — Solutions (continued)

3.



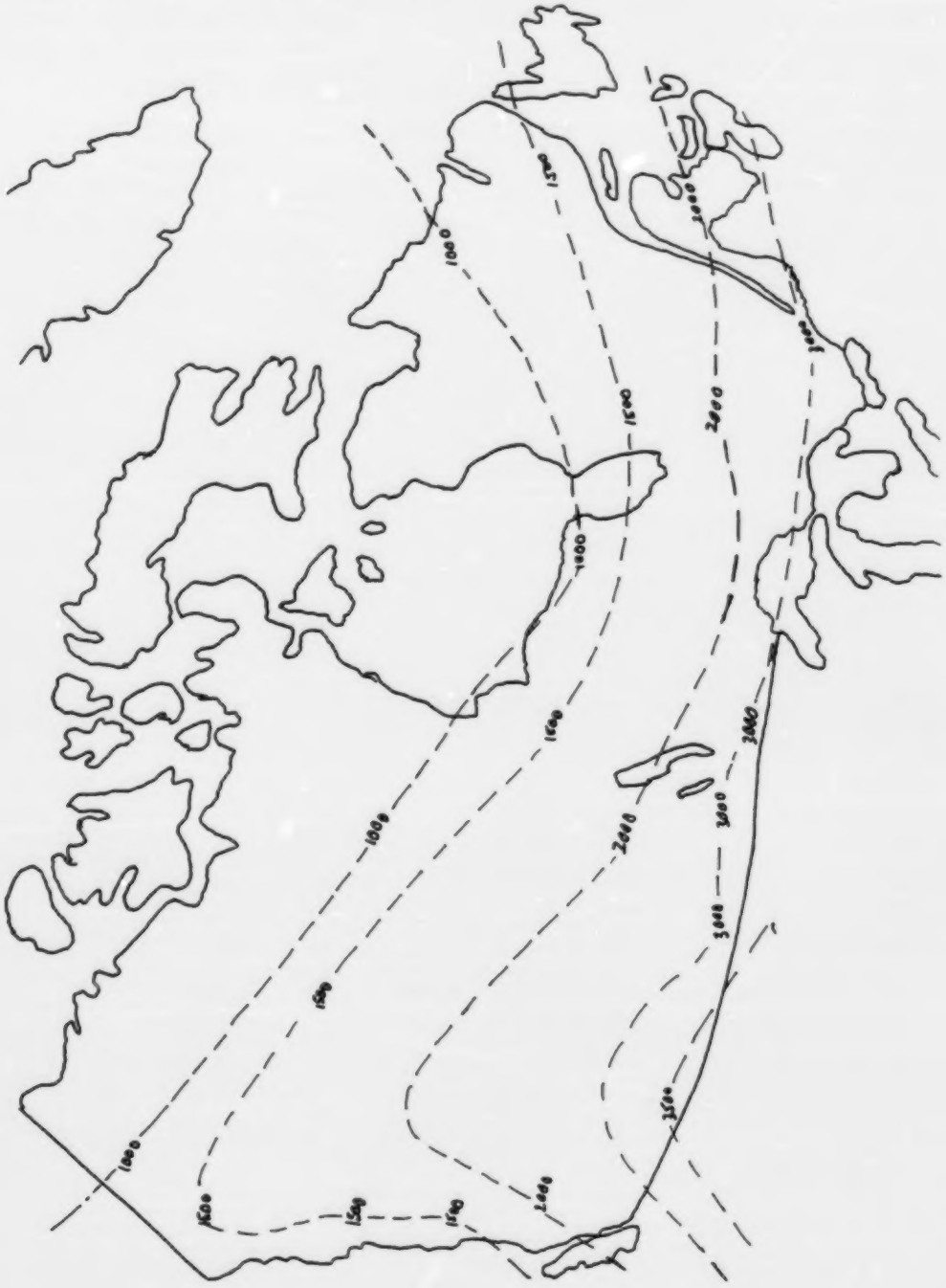
Exercise 5: Contour Maps — Solutions (continued)

4.



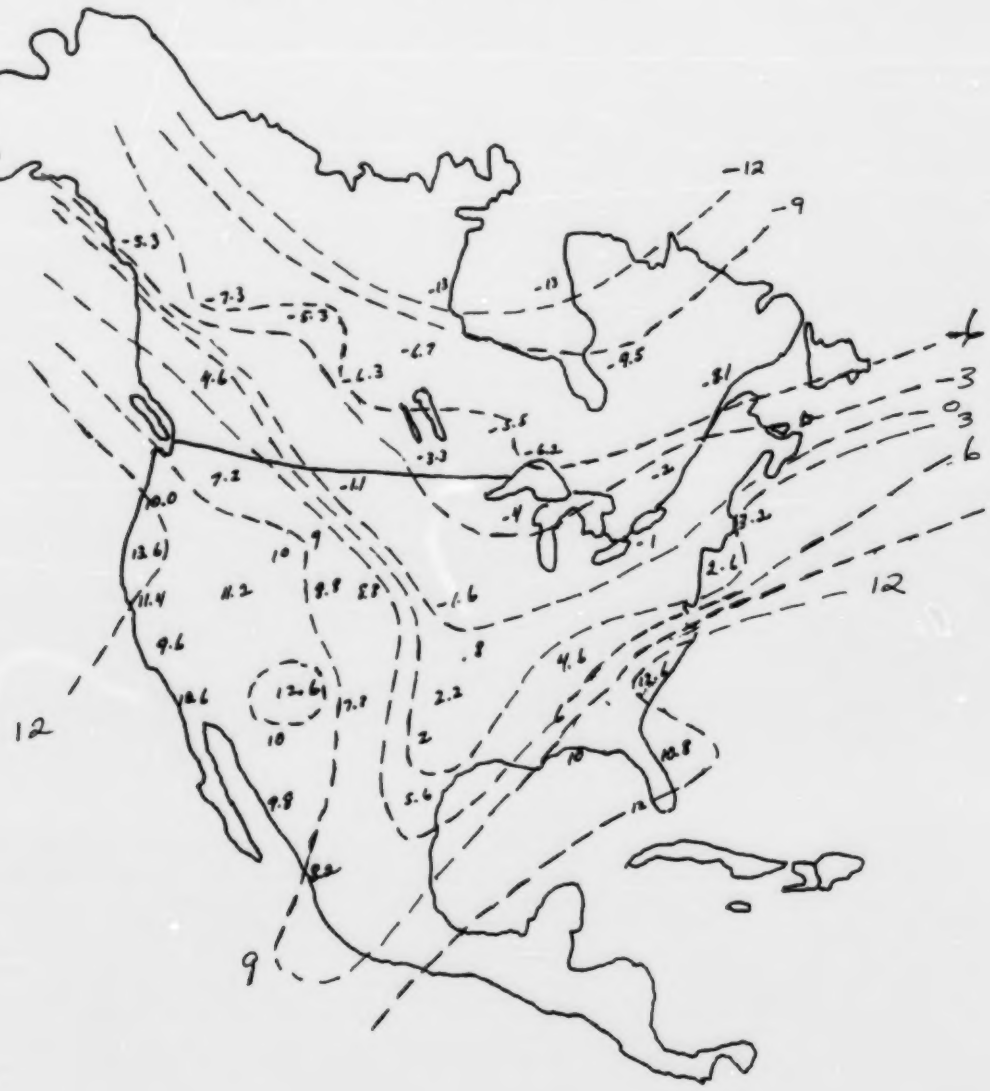
Exercise 5: Contour Maps — Solutions (continued)

5.



Exercise 5: Contour Maps — Solutions (continued)

6.



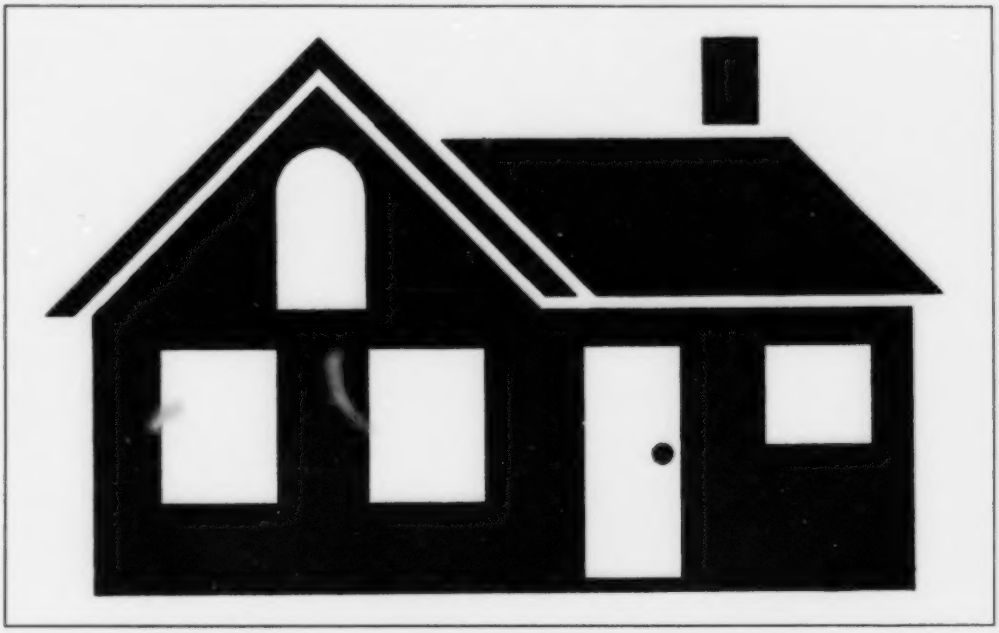
7. Answers will vary.

Unit G
Precision Measurement

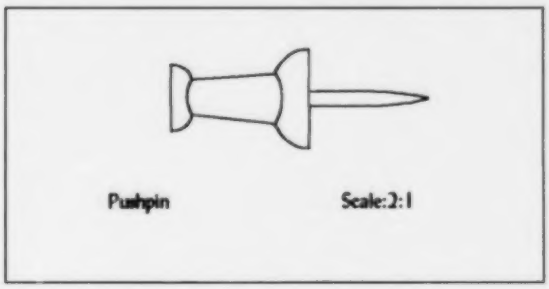
Teachers are advised to refer to *Senior 3 Applied Mathematics: A Foundation for Implementation* as their ***primary*** source of suggestions for instruction and assessment. The following exercises are an excellent resource if supplementary material is required.

Exercise 1: Scaling

1. The house below is drawn at a scale of 1:64 (1" = 5' – 4).
   a) Find the total length of the house.
   b) Find the maximum height of the house.
   c) Draw the house to a scale of 1:48 (1" = 4' – 0).

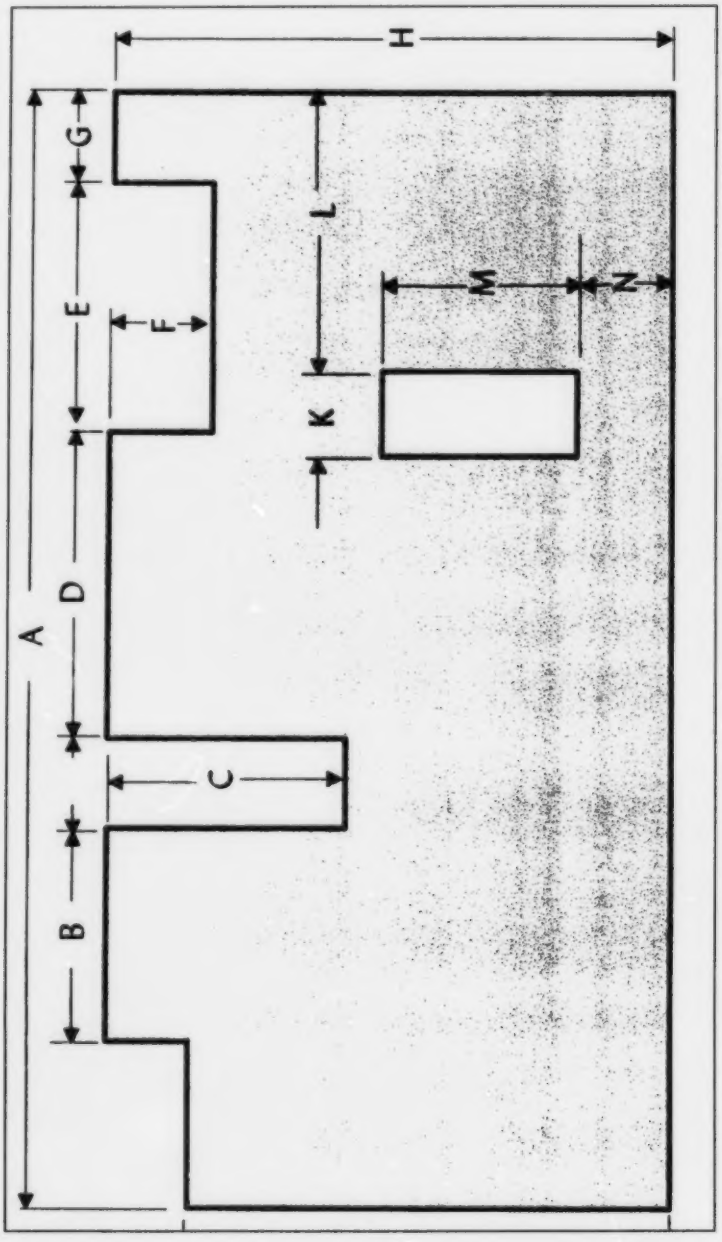


2. Below is a drawing of a pushpin drawn to the scale of 2:1. Redraw the push pin to a scale of 10:1.



Exercise 1: Scaling (continued)

3. Scaling



Note: When measuring, do not measure the dimension lines, measure the object.

a) Using a scale of 1:1, measure the dimensions A through F.

b) Using a scale of 1:2, measure the dimensions G through L (there is no I).

c) Using a scale of 1:5, measure the dimensions M and N.

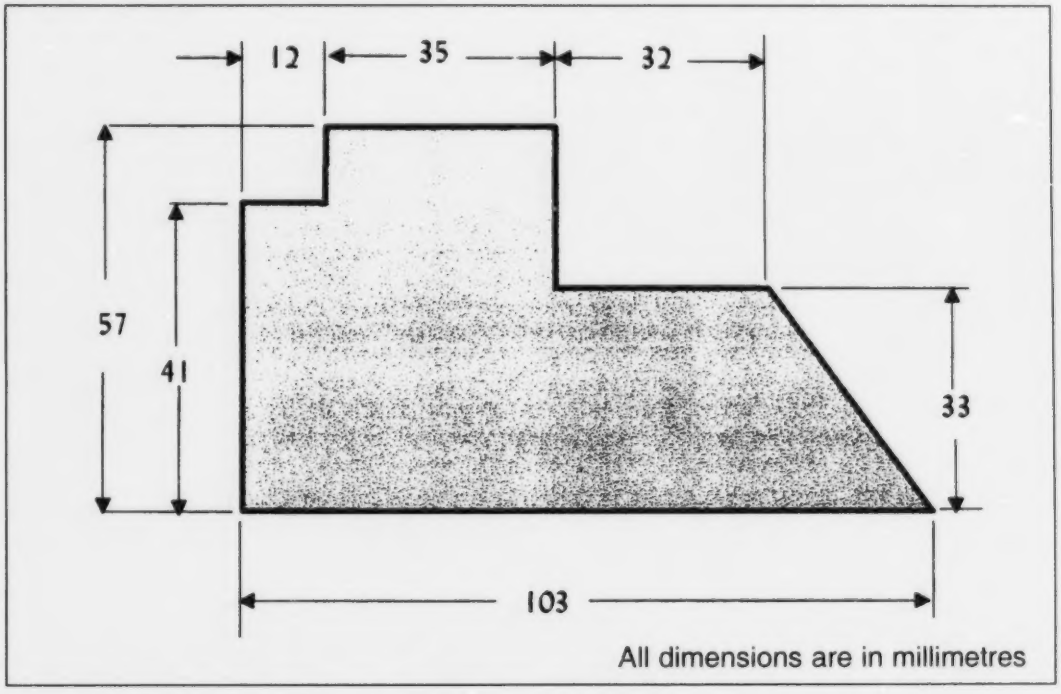
d) Using a scale of 1" = 1' – 0", measure the dimensions A through F.

e) Using a scale of ½" = 1' – 0", measure the dimensions G through L (there is no I).

f) Using a scale of 1½" = 1' – 0", measure the dimensions M and N.

Exercise 1: Scaling (continued)

4. Scaling



a) Redraw the above plate using a scale of 1:2.

b) Redraw the above plate using a scale of 2:1.

c) Calculate the area of the plate in square millimetres.

d) If the plate was cut from material that measures 11 cm x 6 cm, calculate the area of waste material in square millimetres.

Exercise 1: Scaling (continued)

5. The scale ratio of 1:600 000 000 indicates that 1 unit of measure on the plan or map represents 600 000 000 units of actual measurement. If the unit of measure is a millimetre, then the ratio would be 1 mm on the map represents 600 000 000 mm actual. This could also be written as 600 000 metres or 600 kilometres. Another way of showing a scale on a map is 1 cm = 60 km or 1 mm = 6 km; an approximate scale in non-metric units would be 1 inch = 95 statute miles. If you were to take a reading of 50 mm on a map of this scale, the actual distance would be 300 km.

   Map A = 1:600 000 000
   Map B = 1 mm = 6 km

   On Map A, a distance of 52 scaled millimetres is measured.
   On Map B, a distance of 52 scaled millimetres is also measured.

   Compare the two **actual** distances and discuss the implications of precision upon the values each scaled millimetre has on map reading.

6. A room is measured with a steel tape measure whose smallest graduated unit is 1 cm. The length reading is 3 m + 65 cm + (between 65 and 66 cm). The width measurement reads 2 m + 74 cm + (between 74 and 75 cm). Determine the length and width of the room in millimetres. Calculate the area of the room in square metres.

Exercise 2: Presentation

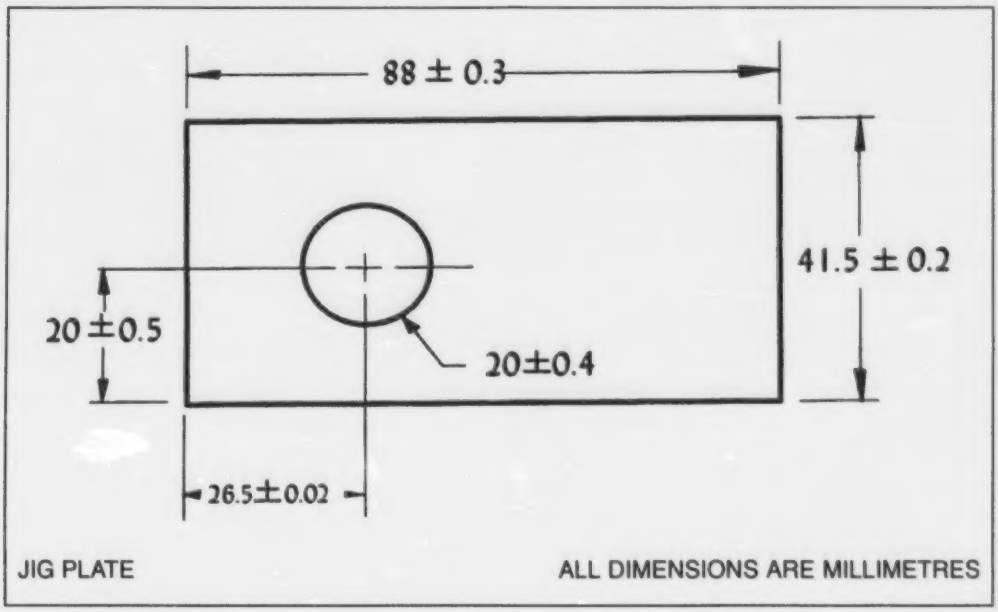
The company you work for makes rectangular plastic food trays and is planning to make new ones with dimensions double those now being made. The company president is convinced that they will need only twice as much plastic as the older tray version. "Twice the size means twice the plastic," she proclaims, and no one is willing to challenge her. Your job and this assignment is to make a presentation to her that will clearly outline the error in thinking and explain in detail why the new trays require eight times as much plastic. Write your presentation including any calculations.

Exercise 3: Greatest Possible Error and Tolerance

1. To what unit of measure are the following measurements precise?
   a) 0.07 cm
   b) 54.8 km
   c) 0.000 5 g
   d) 0.011 mm
   e) 25.8 kg
   f) 5047 L
   g) 4-7/8 in
   h) 25-3/4 h
   i) 9.84 s
   j) 100.5 m
   k) 0.46 mg
   l) 9 kg 10 g
   m) 15 h 45 min
   n) 7 ft 7-1/2 in
2. Which of the following measurements are more precise:
   a) 22.1 m or 10.0 m?
   b) 5-3/16 in or 4-2/3 in?
   c) 4.6 g or 14.23 kg?
   d) 4.93 miles or 9-15/16 yd?
   e) 2-5/6 h or 11 h 40 min?

Exercise 3: Greatest Possible Error and Tolerance (continued)

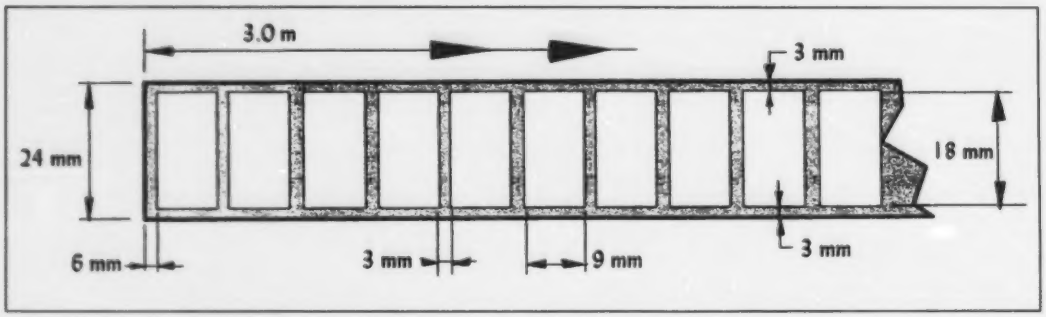
3. Using the drawing of the Jig Plate shown below, determine the following:
   a) The tolerance limits on the
      i) length of the plate
      ii) width of the plate
      iii) diameter of the hole
   b) The percent tolerance on the
      i) length of the plate
      ii) width of the plate
      iii) diameter of the hole
   c) The unit of measurement on the drawing that is most precise
   d) The greatest possible error in each limit dimension on the drawing
   e) If the Jig Plate is made from a material that measures 100 mm x 50 mm and is 10 mm thick, determine the waste of material in cubic centimetres. Write the details of your procedure, including the calculations.



Exercise 4: Maximum and Minimum

1. The area of the opening that a rectangular duct is to be placed on cannot exceed 39 in^2. To maintain an efficient air flow one of the dimensions of the duct must be 12". Calculate the length of the other side of the rectangular duct, as well as the area in square feet of one foot length of the duct.

2. The following strip of metal measuring 24 mm wide by 3 m long is to have rectangular blanks measuring 18 mm x 9 mm punched out of the strip. Starting 6 mm from one end, calculate the maximum number of blanks that can be made from the strip allowing 3 mm between each blank. Determine the amount of waste material in square metres.



3. The internal diameter of a conduit (pipe made to contain a number of conductors, that is, electric wires) is 17.8 mm. What is the maximum number of 4.34 mm diameter conductors the conduit can hold?

4. The horizontal length of the top of a brick wall is 4286 mm. It is to be capped by a number of bricks and joints with the following size: 90 mm brick width and a maximum joint size of 10 mm and a minimum of 8 mm. Calculate the maximum number of bricks and the maximum size of each joint to cap the wall.

5. A housing development is allocated 16.2 ha of land, and 64% of that area is for lots. If each lot is a maximum of 18.3 m x 31.1 m, what is the maximum number of lots to be developed?

6. A rectangular sheet of metal measuring 762 mm x 610 mm has four equal squares cut out at the corners, and the sides are folded to form a rectangular box. What must be the length of the side of each square cut away, so that the volume of the box may be at a maximum of 23 259 cm^3? With the area of the base at a maximum of 207 668 mm^2, find the dimensions of the base of the box in millimetres.

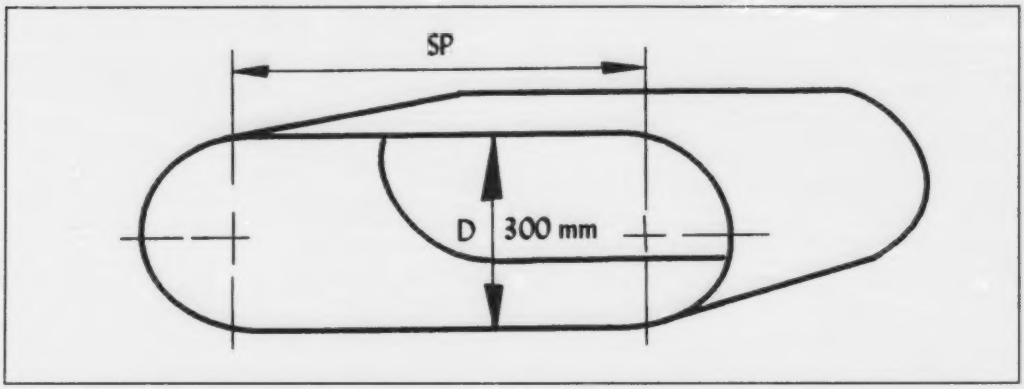
Exercise 5: Tolerances for Area

1. An open tank is to be made of sheet metal. It must have a square base and its sides must be perpendicular to the base. Its capacity is to be 227 000 cm^3. Find the length of the side of the square base and the depth, so that the minimum amount of sheet metal may be used and the height of the tank does not exceed 384 mm. Determine also the area of sheet metal in square centimetres to make the tank.

2. An air duct code requires that the following shape duct have a cross-sectional area of 2400 ± 3.27 cm^2 to allow essential air flow to a specific temperature zone. If the depth of the opening is to be a maximum of 300 mm, find the following:

   a) The maximum and minimum lengths of the perimeter around the duct

   b) The maximum area of material used to make one metre of the duct

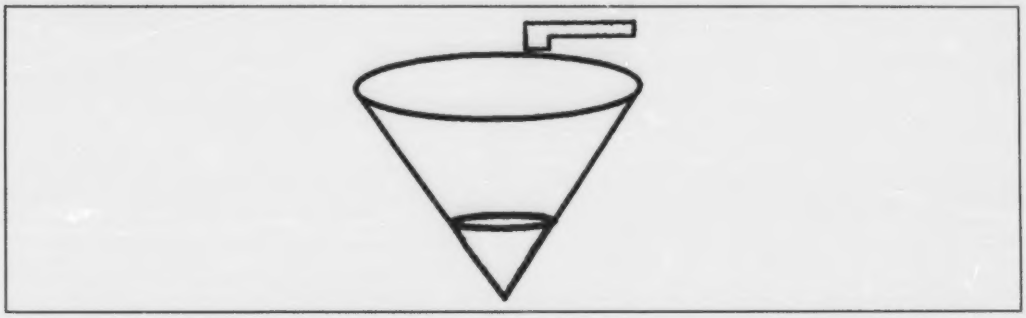


3. A certain electrical box has a capacity of 245 ± 36.87 cm^3. If the usable space required for a particular insulated electrical wire is 24.58 cm^3, calculate the maximum number of insulated wires the box can contain. When the inside of the box measures 101 ± 0.6 mm^2 find the maximum depth of the box in centimetres.

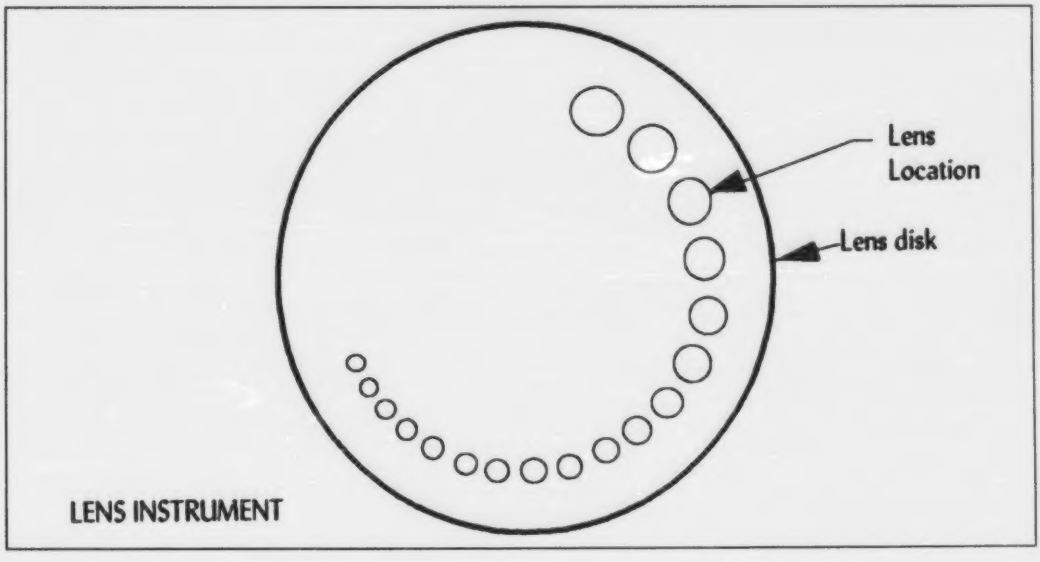
4. A certain flywheel on an engine needs to be lighter. The material of the flywheel has a relative density of 2.70 g/cm^3 and is 127 mm in diameter by 15 mm thick. The flywheel needs to have a maximum of 510 ± 0.739 g. Calculate the size of the diameter drills that are to be used to drill through the flywheel to arrive approximately at both the maximum and minimum weights required.

Exercise 6: Tolerances for Volume

1. Using the volume of a cone as $V_{cone} = \frac{h}{3}(\text{Area of Base})$, apply this to an inverted conical tank that has a top diameter equal to its height and is being filled with water at a rate of $2.00 \pm 0.25\ m^3/h$. With the aid of a spreadsheet program, show the maximum and minimum volume of water in the tank (m^3) and the depth, D, for every 5-minute period up to a maximum of 1 hour.

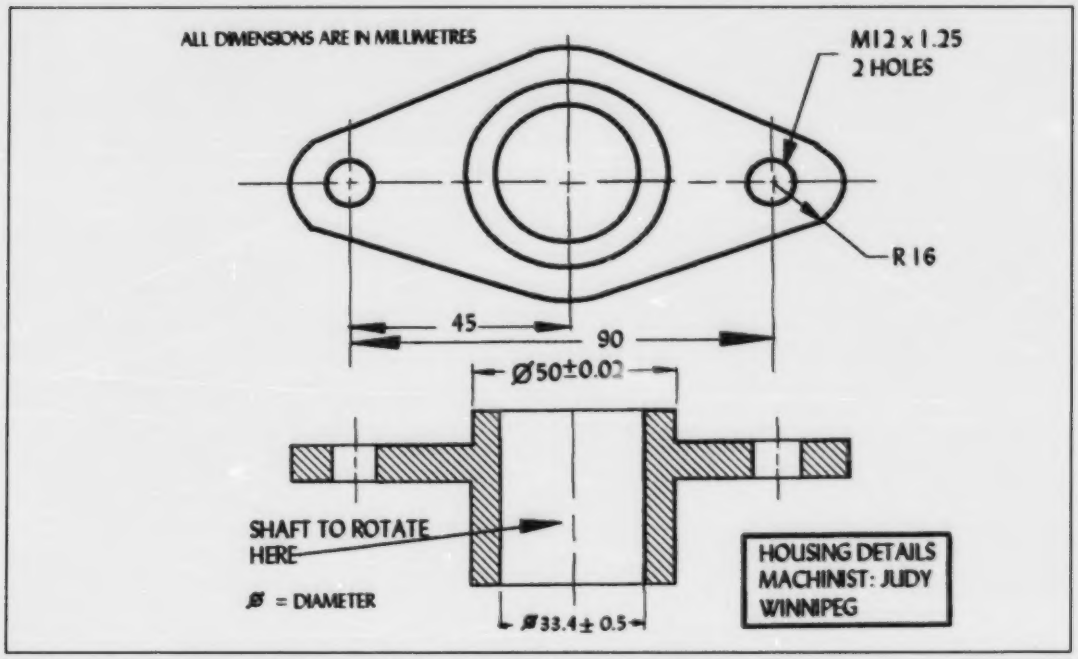


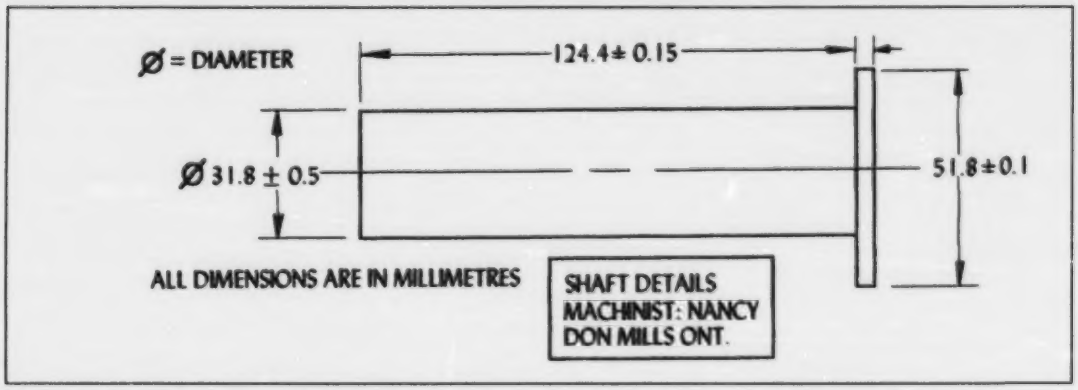
2. A lens in a certain optical instrument has a diameter of 80 mm. This diameter is reduced in a series of circular locations. At each location the area of the opening is reduced by 15% of its previous area. Calculate the required diameter at each location and give the area of each lens in millimetres squared until the diameter is less than 20 mm. Use a spreadsheet program if available.



Exercise 7: Evaluating Mechanisms

Judy is a machinist in Winnipeg who makes bearing housings. Each housing is to support a rotating shaft, but the shafts are made by Nancy in Don Mills, Ontario. To guarantee that the parts will work together, a tolerance is applied to both the housing where the shaft rotates and the shaft diameter.





Note the two limiting conditions applied to the housing and shaft. As the two pieces must function together, the tolerances applied must be given careful consideration.

The ***housing***, where the shaft rotates, is otherwise known as the ***hole***.

Limiting conditions: ϕ 33.4 ± 0.5 mm ϕ = Diameter

Limits: Maximum = 33.9 mm
Minimum = 32.9 mm

Exercise 7: Evaluating Mechanisms (continued)

Shaft:

Limiting conditions: ϕ 31.8 ± 0.5 mm

Limits: Maximum = 32.3 mm
Minimum = 31.3 mm

Allowance, or the intentional differences in correlated dimensions:

Minimum allowance = $M_{min}A$

$M_{min}A$ = Minimum hold limit, 32.9 mm – Maximum shaft limit, 32.3 mm = 0.6 mm

Maximum allowance = $M_{max}A$

$M_{max}A$ = Maximum hole limit, 33.9 mm – Minimum shaft limit, 31.3 mm = 2.6 mm

As the results show, there is a positive allowance between the parts. Both parts are made within the tolerance specified, and function and integrity of each part is maintained.

1. The shaft's limiting conditions are 6 ± 0.02 in. The hole or bearing housing has the limiting condition of 6.08 ± 0.05 in. Calculate the maximum and minimum conditions of allowance that are to prevail.

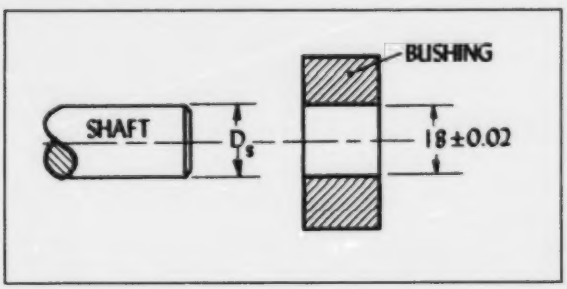
2. The following two parts are to be press fitted together, with no clearance.

   Part A = 40.105 ± 0.015 mm (Shaft)

   Part B = 40.075 ± 0.015 mm (Hole)

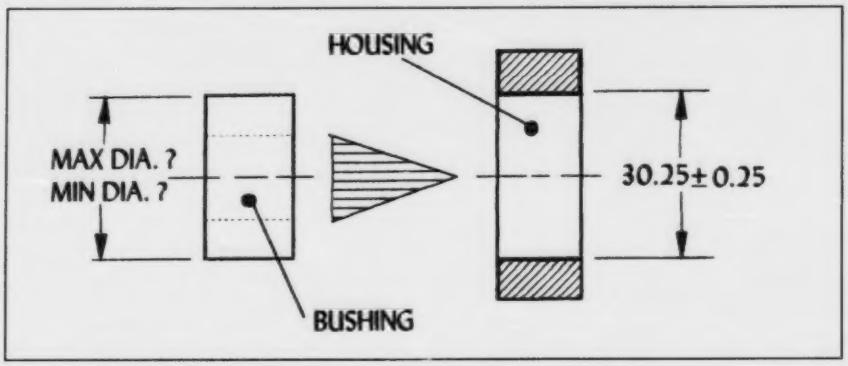
   Calculate, if indeed, the combination does produce an interference fit, and the allowance that results.

3. Below is shown a shaft and bushing assembly. The diameter of the shaft D_s is to have a tolerance of 0.5 mm, and a minimum clearance of 0.2 between the bushing and the shaft. Calculate the limit dimensions of the shaft.



Exercise 7: Evaluating Mechanisms (continued)

4. Below is a diagram of a bushing that is to be press fitted into a housing. The bushing's outside diameter is to have a tolerance of 0.08 mm and a maximum interference of 0.2 mm. Calculate the limit dimensions of the outside diameter of the bushing.



5. Determine the tolerance and the maximum and minimum limits of the following conditions:

   a) 76

   b) 50 ± 0.05

   c) 10

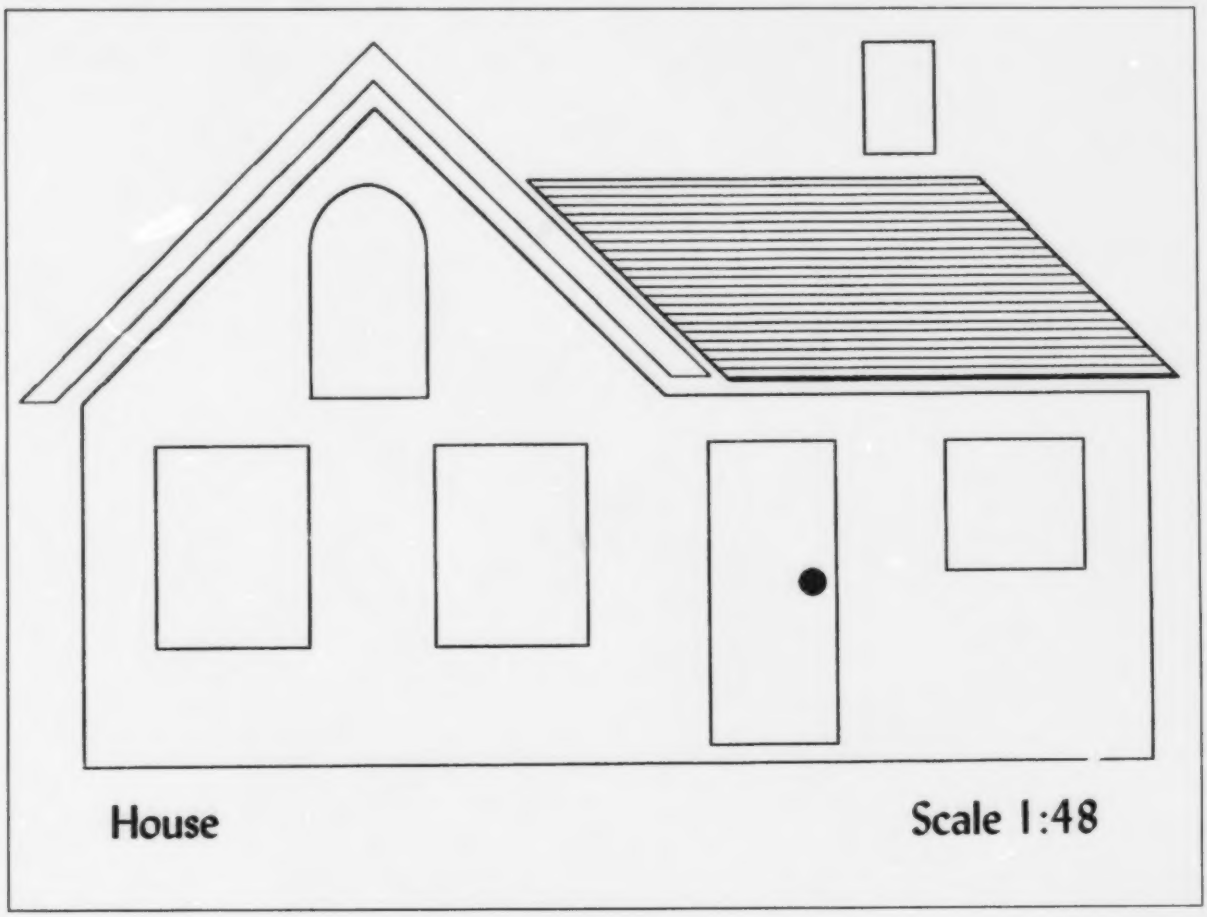
   d) 24.075
      24.060

   e) 90 ± 1.5

Unit G
Precision Measurement Solutions

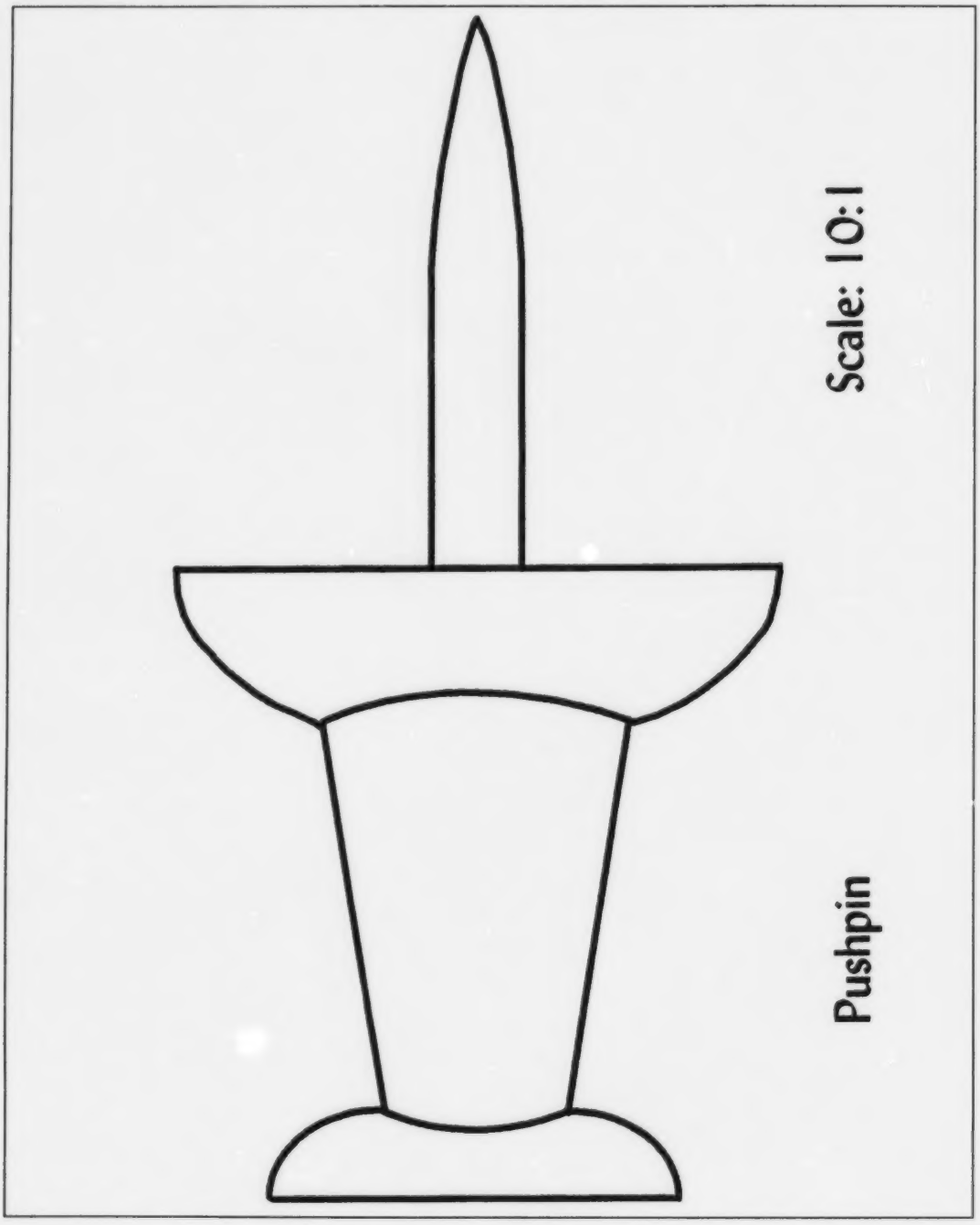
Exercise 1: Scaling — Solutions

1. a) 25' – 9
   b) 17' – 5
   c) See below.



Exercise 1: Scaling — Solutions (continued)

2.



Exercise 1: Scaling — Solutions (continued)

3. a) Scale 1:1

A = 167 mm	B = 32 mm	C = 36 mm
D = 46 mm	E = 36 mm	F = 16 mm

b) Scale 1:2

G = 28 mm	H = 168 mm	I = (no I)
J = 145 mm	K = 22 mm	L = 84 mm

c) Scale 1:5

M = 1450 mm	N = 750 mm

d) Scale 1" = 1' – 0"

A = 6' – 6 3/4"	B = 1' – 3 1/4"	C = 1' – 5 1/4"
D = 1' – 10"	E = 1' – 5 3/4"	F = 7 1/2"

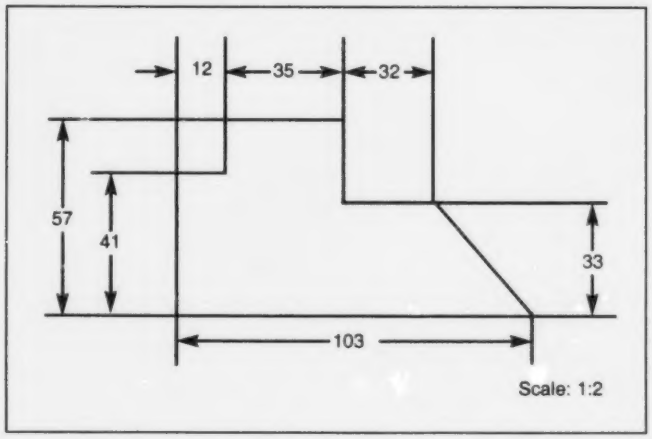
e) Scale 1/2" = 1' – 0"

G = 1' – 1 1/2"	H = 6' – 7"	I = (no I)
J = 5' – 8 1/2"	K = 11 1/2"	L = 3' – 4"

f) Scale 1 1/2" = 1' – 0"

M = 9"	N = 4 3/4"

4. a)



Exercise 1: Scaling — Solutions (continued)

4. b)

Exercise 1: Scaling — Solutions (continued)

4. c) Area of plate with no cuts: $103 \times 57 = 5871\ mm^2$

   Area of cuts: $12 \times 16 = 192$

   $32 \times 24 = 768$

   $(33 \times 24)/2 = 396$

   Gross Area: (Area with no cuts – Area of cuts)

   $5871 - 1356 = 4515\ mm^2$

   d) Area of plate to be cut: $110 \times 60 = 6600\ mm^2$

   Area of cut plate: $= 4515$

   Area of waste: $= 2085\ mm^2$

5. Map A scaled distance measured = 52 mm = actual distance = 3120 km

   Map B scaled distance measured = 52 mm = actual distance = 312 km

   Unit measure comparison: If each millimetre, or part thereof, measured on either map is representative of a specified distance, then the larger the distance represented by a unit of measure the more unreliable the scale used. Precision in locating features on the ground, compared to those on the map, does not depend on the measuring device or the units of graduation. Contours and the Earth's curvatures influence that kind of task.

6. If the smallest unit of measure is the centimetre, then one-half the centimetre is 5 mm.

   Length = 3 m + 65 cm + 5 mm = 3655 mm

   Width = 2 m + 74 cm + 5 mm = 2745 mm

   Area = 3.655 m x 2.745 m = $10.03\ m^2$

Exercise 2: Presentation — Solutions

Student approaches to this subject will vary. The following presentation is meant to be used as a guideline only.

Presentation to
President I.A.M. Hungry
of
Hungry Wait and Eat Services

Synopsis

In the following presentation, it will be clearly seen that Hungry Wait and Eat are on the cutting edge of tray production, and that the latest contract for larger trays will result in an increase in the volume of plastic required. It is also important to compare the amounts of plastic and the appropriate recommendations to offset this increase.

Existing Production Evidence

From last year's production quotas the following data were obtained.

Year of Production	Tray Size	Cost/Tray (Plastic Only)	Amount of Plastic/Tray Volume (cm^3)	Total Produced/Month	Total Cost of Production: Materials + Labour
1998-99	300 x 150 x 5 mm	$11.43	225 ± 0.25 cm^3	8512	$3598.72

Explanation of Table Data

Year of Production: Year ending April 30, 1998 to April 30, 1999

Tray Size: These are out to out dimensions allowing for a recessed base.

Cost of Tray: Average price that fluctuated with stock prices of Sand Dune Plastics Corp. throughout the year.

Amount of Plastic: The tolerance was applied by quality control in allowing for temperature differences in mold extraction. The volume of plastic required for each tray is given by the following formula:

$$V = L \times W \times \text{Thickness}$$
$$= 30 \text{ cm} \times 15 \text{ cm} \times 0.5 \text{ cm}$$
$$= 225 \text{ cm}^3$$

Total Produced: Production numbers were those trays that were shipped. They do not contain the numbers on the production line that were deviations from quality requirements, policies, and test runs.

Exercise 2: Presentation — Solutions (continued)

Cost of Producing 304 Trays/day:

This takes into account the following:

Material:	304 trays/day x $11.43/tray	= $3474.72
Labour:	8 h x $15.50/h	= 124.00
Total Cost:		= $3598.72

New Tray Proposal

Year of Production	Tray Size	Cost/Tray (Plastic Only)	Amount of Plastic/Tray Volume (cm^3)	Total Produced/Month	Total Cost of Production: Materials + Labour
1999-01	600 mm x 300 mm x 10 mm	$91.44	1800 ± 0.50 cm^3	9996	$32 768.08

Explanation:

Cost/Tray: Cost per cubic centimetre of plastic = $11.43/225 cm^3

= $0.0508/$cm^3$

or = 5.08¢/cm^3

Amount of Plastic/Tray

Previous volume: L x W x Thickness

300 x 150 x 5 mm

New tray volume = double size = 600 mm x 300 mm x 10 mm

As each dimension was doubled, the resulting volume was 8 times the original.

For example, if original volume was 1 cm^3 (1 cm x 1 cm x 1 cm)

Doubling the volume would result in (2 m x 2 m x 2 m) = 8 m^3

Therefore, if 225 cm^3 was the original and the new tray size doubled all dimensions, then the new volume would be 225 x 8 cm^3 = 1800 cm^3.

Cost of Production per day:

Material:	357 x 1800 cm^3 x $0.0508/$cm^3$	= $32 644.08
Labour:	8 h x $15.50/h	= 124.00
	Total Cost	= $32 768.08

Conclusion

The above data and details illustrate the volume of plastic required for the new tray product. In addition, they show the minimum cost requirements for a day's production of the new tray. Further, the dimensional findings confirm that when the dimensions of an object are doubled the volume is **eight times** the original volume.

Exercise 3: Greatest Possible Error and Tolerance — Solutions

1. a) 1/100 of a centimetre (1/10 of a millimetre)
   b) 1 hm (1/10 of a kilometre)
   c) 1/10 000 of a gram (1/10 of a milligram)
   d) 1/1000 of a millimetre (1 micrometre); 1 μm
   e) 1/10 of a kilogram (1 hg)
   f) 1 L
   g) 1/8 in
   h) 1/4 h (15 min)
   i) 1/100 of a second
   j) 1/10 of a metre (1 decimetre: 1 dm)
   k) 1/100 of a milligram
   l) 1 g
   m) 1 min
   n) 1/2 in

2. a) Same precision
   b) 5-3/16 in.
   c) 4.6 g
   d) 9-15/16 yds.
   e) 11 h 40 min.

3. a) i) Limits on plate length: Maximum: 88.3 mm; Minimum: 87.7 mm
      ii) Limits on plate width: Maximum: 41.7 mm; Minimum: 41.3 mm
      iii) Limits on hole: Maximum: 20.4 mm; Minimum: 19.6 mm

   b) Note: PT = Percent Tolerance = $\dfrac{\text{Absolute Tolerance}}{\text{Base Dimension}} \times 100$
      i) 0.341%
      ii) 0.482%
      iii) 2.0%

   c) The dimension of the hole from the left-hand side of plate is the most precise. The unit of precision is ± 0.4 mm.

   d) i) The length of the plate: 88 ± 0.3 mm: GPE = 0.05 mm
      ii) The width of the plate: 41.5 ± 0.2 mm: GPE = 0.05 mm
      iii) Hole diameter: 20 ± 0.4 mm: GPE = 0.05 mm
      iv) Vertical position of hole centre: 20 ± 0.5 mm: GPE = 0.05 mm
      v) Horizontal position of hole centre: 26.5 ± 0.02 mm: GPE = 0.005 mm

Exercise 3: Greatest Possible Error and Tolerance — Solutions (continued)

e) Calculations to find the amount of waste material to make the plate:

Volume of plate to be cut = $10 \times 5 \times 1 = 50$ cm^3

Maximum volume of jig plate design

Note: with smallest hole = $\{(8.83 \times 4.17) - (\pi 1.96^2/4)\} \times 1 = 33.80$ cm^3

Minimum waste = $50 - 33.8$ cm^3 = 16.2 cm^3

Minimum volume of jig plate design

Note: with largest hole = $\{(8.77 \times 4.13) - (\pi 2.04^2/4)\} \times 1 = 32.95$ cm^3

Maximum waste = $50 - 32.95$ cm^3 = 17.05 cm^3

Exercise 4: Maximum and Minimum — Solutions

1. Maximum area cross-section of duct $= L \times W$
$= 39 \text{ in}^2 = 12 \times W$
$W = 39/12$
$= 3.25 \text{ in}$

Area of material for one foot length of duct: unit length (ft) x perimeter (ft)
$A = 1\{2(L \times W)\}$
$= 1\{2(1 \times 0.2708)$
$= 0.542 \text{ ft}^2$

2. Maximum number of blanks made = Length of available metal strip ÷ (width + space)
$= (3000 - 6 \text{ mm}) - (9 + 3 \text{ mm})$
$= 2994 \text{ mm} \div 12$
$= 249 \text{ blanks}$
Waste = Area of strip – Area of blanks
$= (3 \times 0.024 \text{ m}) - (249 \times 0.018 \times 0.009 \text{ m})$
$= 0.031\,662 \text{ m}^2$

3. Number of conductors = Area of conduit ÷ Area of conductor
$= (\pi 17.8^2/4) \div (\pi 4.34^2/4)$
$= 17.8^2\mu \div 4.34^2$
$= 16$ conductors (electric wires)

4. Maximum number of bricks = Length of wall ÷ (Size of brick + Maximum joint size)
$= 4286 \div (90 + 10)$
$= 42.86$ bricks and joints
$= 43$ bricks

Let JS = Joint size
Check: $43 \times 90) + 42 \times JS = 4286$
$3870 + 42JS = 4286$
$JS = (4286 - 3870) \div 42$
$= 9.9 \text{ mm}$

5. Number of lots = Area available ÷ Area of lot
$= 64\%(16.2 \text{ ha} \times 10\,000 \text{ m}^2/\text{ha}) \div (18.3 \times 31.1)$
$= 182$ lots

6. Maximum volume = Area of base x height
$23\,259 \text{ cm}^3 = 207\,668 \text{ mm}^2 \times \text{ht}$
$23\,259 \text{ cm}^3 = 2076.68 \text{ cm}^2 \times \text{ht}$
$\text{ht} = 23\,259 \div 2076.68$
$= 11.2$

Size of square cut at the corner = 112 mm x 112 mm
Maximum dimensions of base box $= \{762 - (2 \times 112)\} \times (610 - (2 \times 112)\}$
$= 538 \text{ mm} \times 386 \text{ mm}$

Exercise 5: Tolerances for Area — Solutions

1. Volume required = 227 000 cm^3 = Area of base x Height of tank
   Height of tank cannot exceed 384 mm
   Let S = Side of base

$$\begin{aligned}
\text{Volume required} = 227\,000\ cm^3 &= S^2 \times \text{Height of tank} \\
S^2 &= 227\,000\ cm^3 \div 38.4\ cm \\
&= 5911.458\ cm^2 \\
S &= \sqrt{5911.458} \\
&= 76.9\ cm \\
&= 769\ mm
\end{aligned}$$

Minimum area of material for tank

$$\begin{aligned}
&= \text{Area of base} + \text{Lateral area of tank} \\
&= (76.9^2) + (\text{ht} \times \text{perimeter}) \\
&= (76.9^2) + (38.4 \times 4 \times 76.9) \\
&= 5913.61 + 11\,811.84\ cm^2 \\
&= 17\,725.45\ cm^2
\end{aligned}$$

2. a) Let SP_{max} = Maximum length of straight piece

$$\begin{aligned}
\text{Maximum length of perimeter} &= 2 \times SP_{max} + \pi D \\
\text{Maximum area for duct} &= 2403.27\ cm^2 \\
\text{Maximum length of perimeter} &= 2 \times SP_{max} + \pi 300 \\
\text{Maximum area for duct} &= 2403.27\ cm^2 \\
\pi 30^2/4 + SP_{max} \times 30 &= 2403.27\ cm^2 \\
706.86 + 30SP_{max} &= 2403.27\ cm^2 \\
SP_{max} &= (2403.27 - 706.86) \div 30 \\
SP_{max} &= 56.547\ cm \\
\text{Maximum length of perimeter} &= 2 \times SP_{max} + \pi D \\
&= (2 \times 56.547) + (\pi \times 30) \\
&= 207.342\ cm
\end{aligned}$$

$$\begin{aligned}
\text{Minimum area for duct} &= 2396.73\ cm^2 \\
\text{Let } SP_{min} &= \text{Minimum length of straight piece} \\
\text{Minimum area for duct} &= \pi 30^2/4 + SP_{min} \times 30 \\
\pi 30^2/4 + SP_{min} \times 30 &= 2396.73\ cm^2 \\
706.86 + 30SP_{min} &= 2396.73\ cm^2 \\
SP_{min} &= (2396.73 - 706.86) \div 30 \\
SP_{min} &= 56.329\ cm \\
\text{Minimum length of perimeter} &= 2 \times SP_{max} + \pi D \\
&= (2 \times 56.329) + (\pi \times 30) \\
&= 206.906\ cm
\end{aligned}$$

b) Maximum area of material per metre = Perimeter x 1 m

$$\begin{aligned}
&= 207.342\ cm \times 1\ m \\
&= 2.073\,42\ m \times 1\ m \\
&= 2.07\ m^2
\end{aligned}$$

Exercise 5: Tolerances for Area — Solutions (continued)

3. Let maximum number of insulated wires = N_{iw}

N_{iw} = Cubic capacity of box ÷ Cubic unit of insulated wire

= 281.87 cm^3 ÷ 24.58 cm^3

= 11 wires

Volume of box = L x W x Depth

Let maximum depth of box = D_{max}

As the box is square, the volume will be $L^2 \times D_{max}$

281.87 cm^3 = $10.16^2 \times D_{max}$

D_{max} = 281.87 ÷ 103.2256

= 2.731 cm

4.

Mass of flywheel = Volume x Density

= $\pi D^2/4 \times 1.5 \times 2.7$ g/cm^3

= 513.04 g

Maximum required mass of flywheel = 510.34 g

Mass to be removed = 513.04 g – 510.739 g

= 2.301 g

Volume x Density = Mass

Volume = π(Diameter of drill)$^2/4 \times 1.5$ cm^3

2.301 g = π(Diameter of drill)$^2/4 \times 1.5$ $cm^3 \times 2.7$ g/cm^3

Diameter of Drill = $\sqrt{\{(2.301 \times 4) \div (\pi \times 1.5 \times 2.7)\}}$

= $\sqrt{(0.723\ 388\ 689)}$

= 0.850 cm = 8.5 mm diameter

Minimum required mass of flywheel = 509.261 g

Mass to be removed = 513.04 g – 509.261 g

= 3.779 g

Volume x Density = Mass

Volume = π(Diameter of drill)$^2/4 \times 1.5$ cm^3

2.301 g = π(Diameter of drill)$^2/4 \times 1.5$ $cm^3 \times 2.7$ g/cm^3

Diameter of drill = $\sqrt{\{(3.779 \times 4) \div (\pi \times 1.5 \times 2.7)\}}$

= $\sqrt{(1.188\ 042\ 528)}$

= 1.089 973 637 cm = 10.9 mm diameter

Exercise 6: Tolerances for Volume — Solutions

1. Maximum volume of cone in 1 hour = 2.25 m^3
   Minimum volume of cone in 1 hour = 1.75 m^3
   Spreadsheet formulas and approach:

$$V = \frac{h}{3}(\text{Area of base})$$

Base diameter is equal to its height ($h = d$)

Maximum volume in one hour:

$$\frac{h}{3}(\pi h^2) = 2.25 \text{ m}^3$$

$$\frac{\pi h^3}{3} = 2.25 \text{ m}^3$$

$$h = \sqrt[3]{\left\{\frac{(2.25 \times 3)}{\pi}\right\}}$$

$$= 1.29 \text{ m}$$

Maximum volume in 5 min:
2.25 m^3 = 60 min
Divide both sides by 12:

0.1875 m^3 = 5 min

Depth of tank at 5 min:

$$\text{Volume} = \frac{h}{3}(\text{Area of base})$$

$$\frac{h}{3}(\pi h^2) = 0.1875 \text{ m}^3$$

$$\frac{\pi h^3}{3} = 0.1875 \text{ m}^3$$

$$h = \sqrt[3]{\left\{\frac{(0.1875 \times 3)}{\pi}\right\}}$$

$$= 0.564 \text{ m}$$

Depth of water in 5 min is 0.564 m with a maximum volume of 0.1875 m^3.
Repeat procedure using appropriate values for minimum volumes of water for every 5-minute period.

Exercise 6: Tolerances for Volume — Solutions

2. Area of first lens $= \pi D^2/4 = (\pi \times 802) \div 4$
$= 5026.548 \text{ mm}^2$ with a diameter of 80 mm

Diameter $= \sqrt{\{(\text{Area} \times 4) \div \pi\}}$

Subsequent lenses:

$5026.548 \text{ mm}^2 - 15\% = 4\,272.566 \text{ mm}^2$;		Diameter = 73.7 mm
$4272.566 \text{ mm}^2 - 15\% = 3\,631.811 \text{ mm}^2$;		Diameter = 68.0 mm
$3631.811 \text{ mm}^2 - 15\% = 3\,087.394 \text{ mm}^2$;		Diameter = 62.7 mm
$3087.394 \text{ mm}^2 - 15\% = 2\,624.285 \text{ mm}^2$;		Diameter = 57.8 mm
$2624.285 \text{ mm}^2 - 15\% = 2\,230.642 \text{ mm}^2$;		Diameter = 53.3 mm
$2230.642 \text{ mm}^2 - 15\% = 1\,896.046 \text{ mm}^2$;		Diameter = 49.1 mm
$1896.046 \text{ mm}^2 - 15\% = 1\,611.639 \text{ mm}^2$;		Diameter = 45.3 mm
$1611.639 \text{ mm}^2 - 15\% = 1\,369.893 \text{ mm}^2$;		Diameter = 41.8 mm
$1369.893 \text{ mm}^2 - 15\% = 1\,164.409 \text{ mm}^2$;		Diameter = 38.5 mm
$1164.409 \text{ mm}^2 - 15\% = 989.748 \text{ mm}^2$;		Diameter = 35.5 mm
$989.784 \text{ mm}^2 - 15\% = 841.316 \text{ mm}^2$;		Diameter = 32.7 mm
$841.316 \text{ mm}^2 - 15\% = 715.119 \text{ mm}^2$;		Diameter = 30.2 mm
$715.119 \text{ mm}^2 - 15\% = 607.851 \text{ mm}^2$;		Diameter = 27.8 mm
$607.851 \text{ mm}^2 - 15\% = 516.673 \text{ mm}^2$;		Diameter = 25.6 mm
$516.673 \text{ mm}^2 - 15\% = 439.172 \text{ mm}^2$;		Diameter = 23.6 mm
$439.172 \text{ mm}^2 - 15\% = 373.296 \text{ mm}^2$;		Diameter = 21.8 mm
$373.296 \text{ mm}^2 - 15\% = 317.302 \text{ mm}^2$;		Diameter = 20.1 mm
$317.302 \text{ mm}^2 - 15\% = 269.707 \text{ mm}^2$;		Diameter = 18.5 mm

Exercise 7: Evaluating Mechanisms — Solutions

1. Minimum Allowance = 0.01 in. (smallest hole – largest shaft)
   Maximum Allowance = 0.15 in. (largest hole – smallest shaft)

2. Minimum Allowance = –0.06 mm (interference) (smallest hole – largest shaft)
   Maximum Allowance = –0.03 mm (interference) (largest hole – smallest shaft)

3. Minimum clearance (allowance) = Minimum Bushing diameter – Maximum shaft diameter

$$\begin{aligned} 0.2 &= 17.98 - \text{Max } D_s \\ \text{Max } D_s &= 17.98 - 0.2 \\ &= 17.78 \text{ mm} \end{aligned}$$

$$\begin{aligned} \text{Tolerance} &= \text{Maximum limit} - \text{Minimum limit} \\ 0.5 &= 17.78 - \text{Minimum limit} \\ \text{Min } D_s &= 17.78 - 0.5 \\ &= 17.58 \text{ mm} \end{aligned}$$

   Maximum shaft diameter = 17.78 mm
   Minimum shaft diameter = 17.58 mm

4. ID = Internal Diameter; OD = Outside Diameter

$$\begin{aligned} \text{Maximum interference} &= \text{Minimum housing ID} - \text{Maximum bushing OD} \\ -0.2 &= 30.00 - \text{Maximum } OD_{\text{Bushing}} \\ \therefore \text{Maximum } OD_{\text{Bushing}} &= 30.00 - (-0.2) \\ &= 30.20 \text{ mm} \end{aligned}$$

$$\begin{aligned} \text{Tolerance} &= \text{Maximum limit} - \text{Minimum limit} \\ 0.08 &= 30.20 - \text{Minimum limit} \\ \therefore \text{Minimum } OD_{\text{Bushing}} &= 30.20 - 008 \\ &= 30.12 \text{ mm} \end{aligned}$$

5. a) Tolerance = 0.76; Maximum limit = 76.00, Minimum limit = 75.24
   b) Tolerance = 0.10; Maximum limit = 50.05, Minimum limit = 49.95
   c) Tolerance = 0.02; Maximum limit = 10.02, Minimum limit = 10.00
   d) Tolerance = 0.015; Maximum limit = 24.075, Minimum limit = 24.060
   e) Tolerance = 3.0; Maximum limit = 91.5, Minimum limit = 88.5

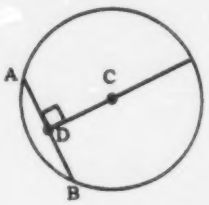
Unit H
Geometry

Teachers are advised to refer to *Senior 3 Applied Mathematics: A Foundation for Implementation* as their ***primary*** source of suggestions for instruction and assessment. The following exercises are an excellent resource if supplementary material is required.

Exercise 1: Exploring Circles and Polygons

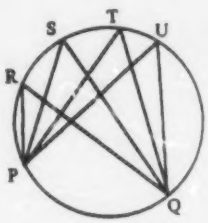
The following activities explore the properties of circles and polygons. Complete these exercises using geometry software.

1. a) Construct a circle with centre C.
   b) Construct two points on the circle, and label them A and B.
   c) Join the points, forming chord AB.
   d) Construct the perpendicular through the centre C, to chord AB.
   e) Construct the point of intersection at D.
   f) Construct AD and BD. (***Note:*** This step is necessary for *Cabri Geometry*, but not for *Geometer's Sketchpad*.)
   g) Measure AD and BD. What do you notice?
   h) Select point A and move along the circumference. What do you notice about AD and BD?

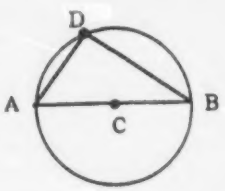


2. a) Construct a circle with centre C.
   b) Construct three points on the circle, P, Q, and R.
   c) Construct chords PR and QR, and radii PC and QC.
   d) Mark and measure angles ∠ PRQ and ∠ PCQ.
   e) What do you notice about the measures of the two angles?
   f) Select point P and move along the circumference. What do you notice about the angles?

3. a) Construct a circle and two points on the circle, P and Q.
   b) Construct and label four points, R, S, T, and U on major arc PQ.
   c) Construct line segments from P and Q to point R, forming ∠ PRQ. Similarly, construct ∠ PSQ, ∠ PTQ, and ∠ PUQ.
   d) These inscribed angles are all subtended by the same arc, PQ. Make each angle a different colour. Mark and measure each angle.
   e) Examine the measure of each angle. What do you notice?

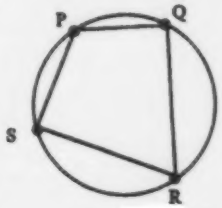


4. a) Construct a circle with centre C.
   b) Construct diameter AB. (Ensure that the diameter actually intersects centre C.)
   c) Construct a point D on the circle.
   d) Construct line segments AD and BD.
   e) Mark and measure ∠ ADB. What do you notice?
   f) Select point D and move along circumference. What do you notice?

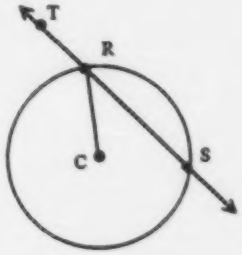


Exercise 1: Exploring Circles and Polygons (continued)

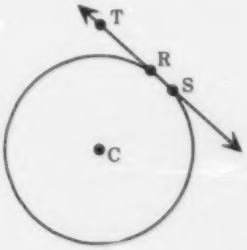
5. a) Construct a circle. Construct four points, P, Q, R, and S, on the circle.
   b) Construct chords PQ, QR, RS, and SP, forming quadrilateral PQRS.
   c) Mark and measure each angle of the quadrilateral.
   d) Find the sums of angles P and R, and angles Q and S. What do you notice?
   e) Move the vertices around the circumference, forming different-shaped quadrilaterals. Find the sums of the pairs of opposite angles. What do you notice?



6. a) Construct a circle with centre C.
   b) Construct a secant line through the circle.
   c) Construct the points of intersection, R and S, of the secant line with the circle, and colour them red.
   d) Construct radius CR.
   e) Construct a point T on the line on the exterior of the circle.
   f) Mark and measure ∠CRT.

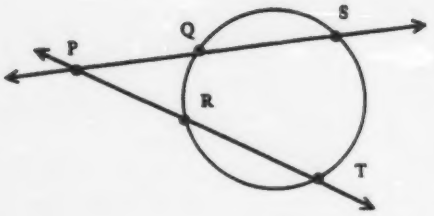


   g) Drag the line slowly towards the end of the circle until R and S converge at a single point. This is the tangent line.
   h) Examine CRT. What is its measure? What conclusion can you make?

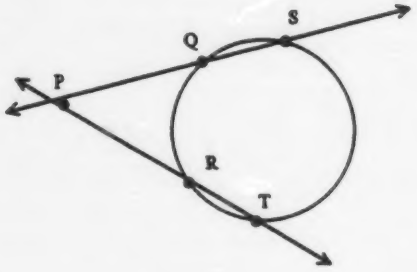


Exercise 1: Exploring Circles and Polygons (continued)

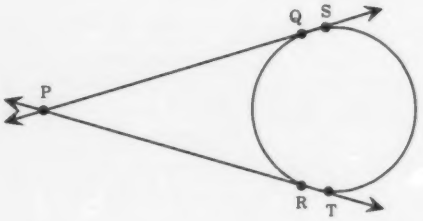
7. a) Construct a circle.
   b) Construct a point P on the exterior of the circle.
   c) Construct two secant lines through P.
   d) Construct the points of intersection of the secant lines with the circle. Label them Q, R, S, and T, as shown.
   e) Construct and measure line segments PQ and PR.



   f) Drag each secant line towards the edge of the circle until Q and S converge, and R and T converge, becoming tangent lines.
   g) What do you notice about PQ and PR?



   h) Drag point P to another position on the exterior of the circle. Repeat step (f), forming tangent lines. Are PQ and PR still congruent?

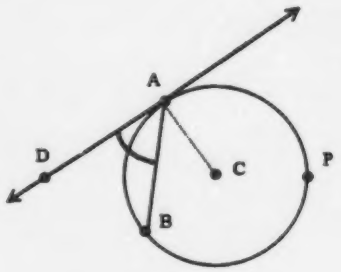


Alternative Method

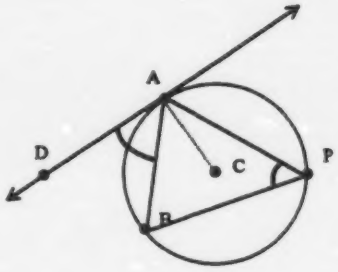
a) Construct a circle with centre C, and radii CR and CQ.
b) Construct perpendicular lines to CR at R, and to CQ at Q. These are tangent lines.
c) Construct the intersection of the two tangent lines. Label this point P.
d) Construct and measure line segments PQ and PR. What do you notice?
e) Drag either point Q or point R along the circle. What do you notice about PQ and PR?

Exercise 1: Exploring Circles and Polygons (continued)

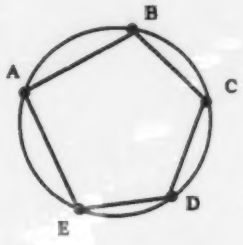
8. a) Draw a circle with centre C and radius AC.
   b) Construct the perpendicular to AC at A. This is a tangent line.
   c) Construct a point B on the circle.
   d) Construct chord AB.
   e) Construct a point D on the tangent line.
   f) Mark and measure ∠ DAB.
   g) Construct another point P on the circle, on the exterior of ∠ DAB.

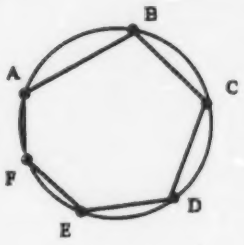


   h) Construct chords AP and PB.
   i) Mark and measure the inscribed angle ∠ APB.
   j) Compare the measures of ∠ DAB and ∠ APB. What do you notice?
   k) Move point B along the circumference. Compare the measures of ∠ DAB and ∠ APB. Are they still equal?



9. a) Construct a circle.
   b) Construct five points, A, B, C, D, and E on the circle.
   c) Construct five chords, joining consecutive points, forming pentagon ABCDE.
   d) Mark and measure each of the five angles.
   e) Find the sum of the five angles.
   f) Compare your answer to the expression $180(n - 2)°$ where n is 5, the number of sides of the pentagon. Are the two values equal?
   g) Add a sixth point, F, to the circle, and construct a hexagon. Repeat the steps above for the hexagon. Is the expression true for a hexagon?
   h) Add points to create other n-gons.

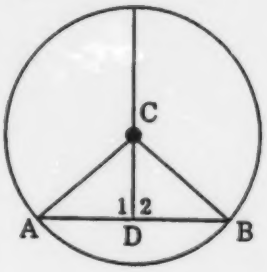




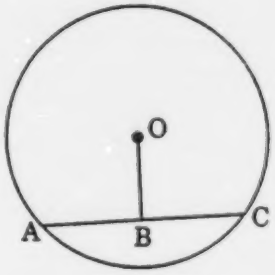
Exercise 2: Circle and Polygon Investigations

Do the following problems after learning the properties of circles and polygons.

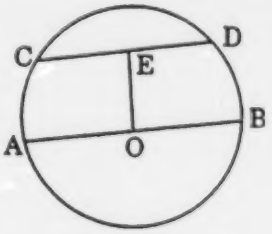
1. a) In circle centre C, D is the midpoint of any chord AB. What can you conclude about AC and BC?
   b) What can you conclude about Δ ADC and Δ BDC?
   c) Why is DC perpendicular to AB?
   d) Will the centre of a circle always lie on the perpendicular bisector of a chord? Explain.



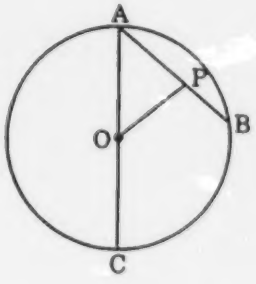
2. This circle has a centre at O, and OB ⊥ AC, with OB = 4 and BC = 3.
   a) Find the length of AB.
   b) Find the length of AC.
   c) Find the length of the radius of the circle.
   d) Find the length of the diameter of the circle.



3. This circle has a centre at O, with OE ⊥ CD, AB = 12, and CD = 8.
   a) Find the length of OD.
   b) Find the length of CE.
   c) Find the length of OE.

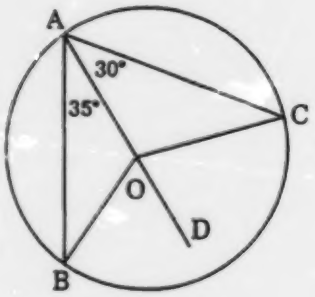


4. This circle has a centre at O, OP ⊥ AB, AC = 16, and AP = OP.
   a) Find the length of OP.
   b) Find the length of OC.
   c) Find the length of AB.
   d) Find the length of AP.

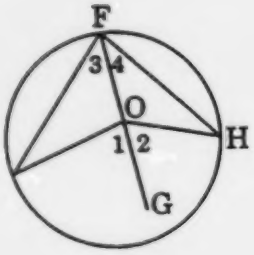


Exercise 2: Circle and Polygon Investigations (continued)

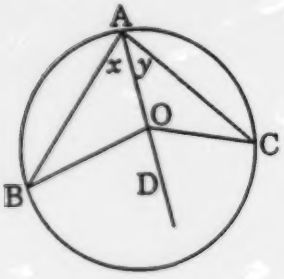
5. O is the centre of the circle.
   a) Find the measure of ∠ BOD.
   b) Find the measure of ∠COD.
   c) What is the relationship between the measures of ∠ BAC and ∠ BOC?



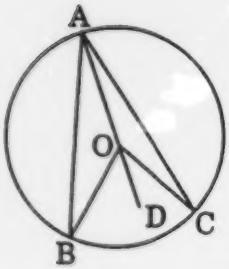
6. This circle has centre at O, with ∠ 1 = 44° and ∠ 2 = 98°.
   a) Find the measures of ∠ 3 and ∠ 4.
   b) What is the relationship between the measures of ∠ GFH and ∠ GOH?



7. This circle has centre at O.
   a) Find ∠ BOD in terms of x.
   b) Find ∠ COD in terms of y.
   c) What is the measure of ∠ BAC?
   d) What is the measure of ∠ BOC?

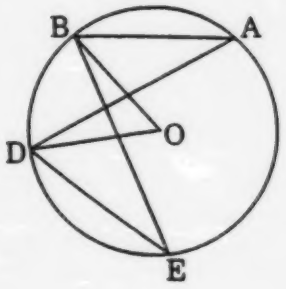


8. This circle has centre at O, with ∠ BAO = 20° and ∠ CAO = 15°.
   a) Find the measure of ∠ BOD.



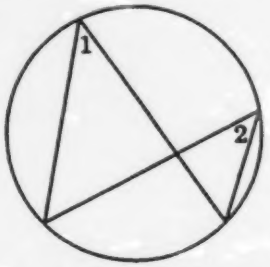
Exercise 2: Circle and Polygon Investigations (continued)

9. This circle has centre at O, and $\angle A = 27°$.
   a) What is the measure of $\angle BOD$ and why?

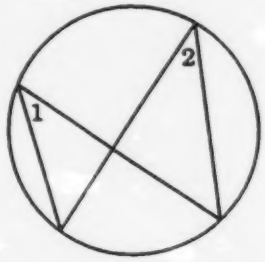


10. Using a protractor, measure $\angle 1$ and $\angle 2$ for each of the circles below. What did you discover?

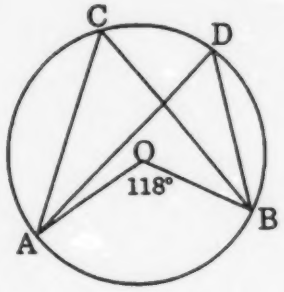
a)



b)



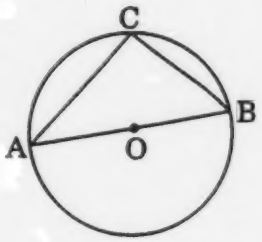
11. The centre of this circle is at O.
   a) What is the measure of $\angle$ C? Why?
   b) What is the measure of $\angle$ D?



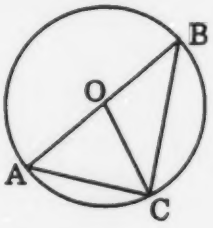
12. Construct a circle with centre at O. Draw a diameter AB. Plot a point C anywhere on the circle. Construct line segments AC and BC.
   a) What is the measure of the angle at C?
   b) If AC = 5 and BC = 12, then find the length of AB.
   c) What is the length of OC?

Exercise 2: Circle and Polygon Investigations (continued)

13. This circle has centre at O, with OB = 5 and BC = 6.
    a) Find the length of AB.
    b) Find the length of AC.

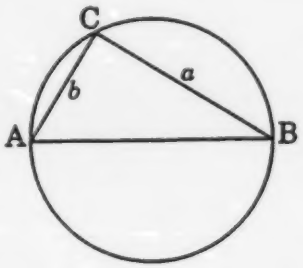


14. This circle has centre at O, with AC = 5 and OC = 6.5.
    a) Find the length of AB.
    b) Find the length of BC.
    c) Find the area of Δ ABC.
    d) Find the area of the circle.



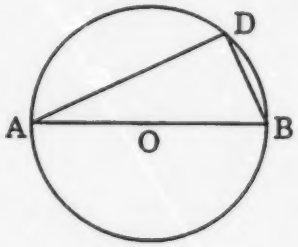
15. Given that AB is the diameter of this circle, prove that the area of the circle is given by

$$\text{Area} = \pi\left(\frac{a^2+b^2}{4}\right).$$



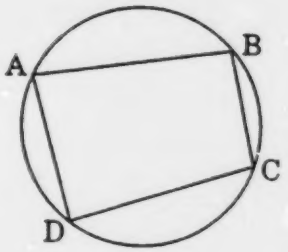
16. a) Construct a circle of radius approximately 4 cm.
    b) Construct a diameter of the circle and label the end points A and B.
    c) Draw a point C anywhere on the circumference of the circle.
    d) Complete the triangle ACB.
    e) Determine the measure of ∠ ACB.

17. If AB is a diameter of this circle, and O is the centre,
    a) what is the measure of ∠ AOB?
    b) what is the measure of ∠ D? Explain why.

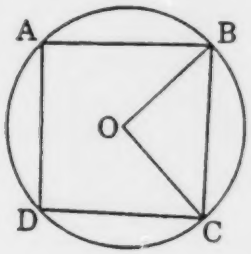


Exercise 2: Circle and Polygon Investigations (continued)

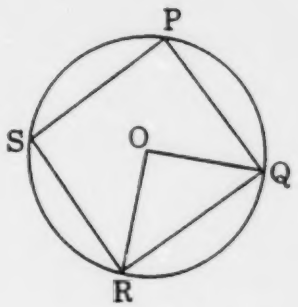
18. Given: Inscribed quadrilateral ABCD
    $\angle A = 68°$
    $\angle D = 2\angle B$
    a) Find $\angle C$.
    b) Find $\angle B$.
    c) Find $\angle D$.



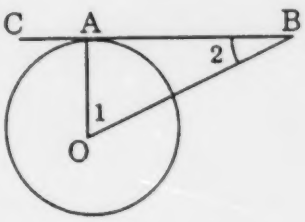
19. Given: Circle with centre O
    $\angle DCO = 30°$
    $\angle ABO = 20°$
    $\angle BOC = 100°$
    a) Find $\angle ABC$.
    b) Find $\angle A$.
    c) Find $\angle D$.



20. Given: Circle with centre O
    ΔOQR is equilateral
    $\angle ORS = 35°$
    $\angle PQO = 28°$
    a) Find $\angle ORQ$
    b) Find $\angle PQR$
    c) Find $\angle S$.
    d) Find $\angle P$.

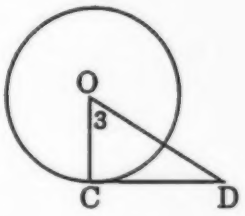


21. Given the circle with centre O, with AB tangent at A, and $\angle 1 = 49°$.
    Find $\angle 2$.

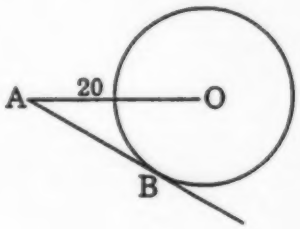


Exercise 2: Circle and Polygon Investigations (continued)

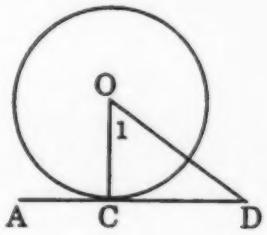
22. Given circle with centre at O, CD tangent to C, and OC = DC, find ∠ 3.



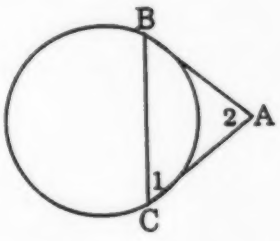
23. Given circle with centre at O, radius = 5, the distance OA is 20, and the line through A is tangent at B, determine the distance AB.



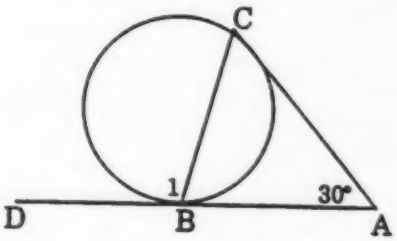
24. Given circle with centre O, AD tangent to C, and ∠ ODC = 40°, verify ∠ 1 = 50°.



25. Given AB and AC are tangent at B and C, and ∠ 1 = 40°, find ∠ 2.

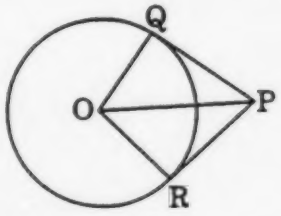


26. Given that AD and AC are tangents at B and C, and ∠ A = 30°, find ∠ 1.



Exercise 2: Circle and Polygon Investigations (continued)

27. Given circle with centre O, tangent segments PQ and PR at Q and R, radius = 8, and QP = 15, find the distances OP, OR, and RP.



28. Two tangent segments to a circle from a point in the exterior form an angle of 60°. If the diameter of the circle is 10, how long are the tangent segments?

29. A polygon has 10 sides. What is the sum of the interior angles?

30. A polygon has 14 sides. What is the sum of the interior angles?

31. A polygon has 102 sides. What is the sum of the interior angles?

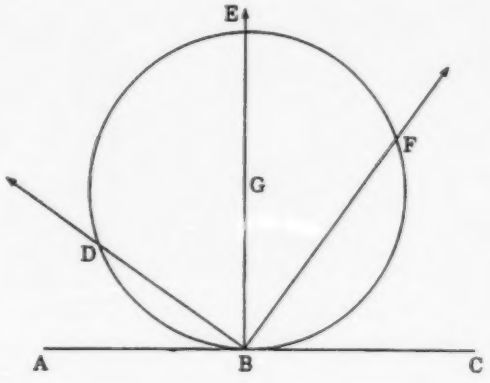
32. A polygon has n sides. What is the sum of the interior angles?

33. The sum of the interior angles of a polygon is 1080°. How many sides does the polygon have?

34. The sum of the interior angles of a polygon is 4500°. How many sides does the polygon have?

35. The sum of the interior angles of a polygon is S°. In terms of S, how many sides does the polygon have?

36. Given: AC is a tangent to circle G at B
    BE is a diameter
    BD ⊥ BF
    $\overset{\frown}{FE} = 78°$

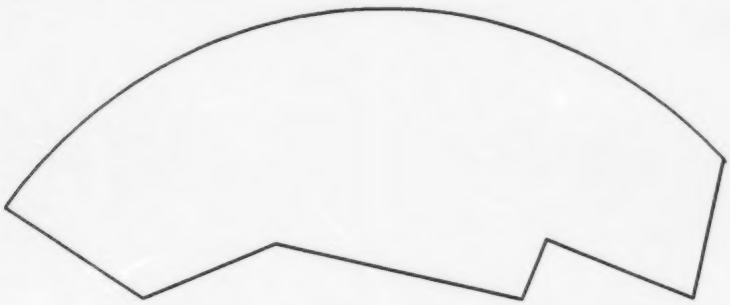


a) Find ∠ EBF.
b) Find $\overset{\frown}{FB}$.
c) Find ∠ FBC.
d) Find $\overset{\frown}{ED}$.
e) Find $\overset{\frown}{BD}$.
f) Find ∠ DBE.
g) Find ∠ ABD.

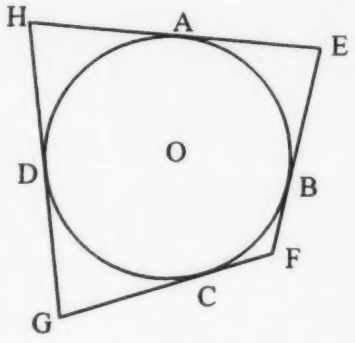
Exercise 3: Problem Solving Using Circle and Polygon Properties

In the following exercises the properties of circles and polygons are used to solve a variety of problems, including design and layout problems. In some cases, a geometry software program is required to solve the problem.

1. If n points are located on the circumference of a circle, how many chords may be formed by joining pairs of points in as many ways as possible? Draw diagrams to determine the number of chords for $n = 2, 3, 4, 5, 6, 7$. Find a pattern and make a conjecture.
   a) Use logical reasoning to find the number of chords formed by 100 points.
   b) Generate a formula for n points. A graphing calculator may be used to generate the formula.
2. If n points are located on the circumference of a circle and chords are drawn to join the points, find the maximum number of regions that can be formed. Create a table of values showing the relation between the maximum number of regions and the number of points on the circumference. Determine the obvious relation (formula) for $n = 1, 2, 3, 4$, or 5. Show that the obvious relation does not work for $n = 6$.
3. The diagram shows a part of a broken plate. What was the diameter of the plate? Describe how you would determine the diameter.

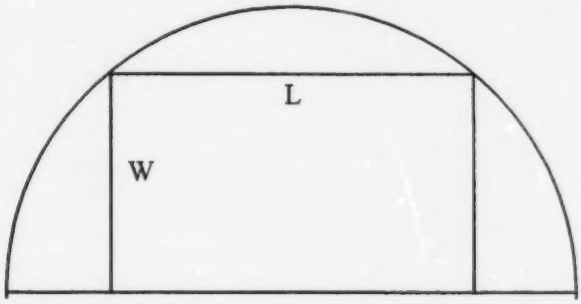


4. Draw three overlapping circles so that there is a region common to the interiors of all three circles. Select any two of the circles and draw the common chord. Repeat this two more times until all three common chords have been drawn. What property about these three chords will always be true?
5. Draw a circle with centre O. Mark four points A, B, C, and D on the circle, and draw four tangent lines at these four points as shown. Label the points of intersection of the four tangent lines E, F, G, and H. EFGH should now be a quadrilateral. Measure the lengths of the sides of quadrilateral EFGH. Now drag one or more of the points A, B, C, or D along the circle and measure the side lengths again. The lengths of the sides of a quadrilateral drawn in this way have a certain property. What is it?

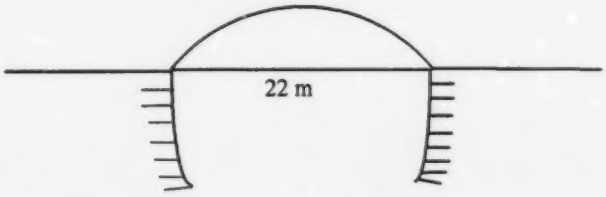


Exercise 3: Problem Solving Using Circle and Polygon Properties (continued)

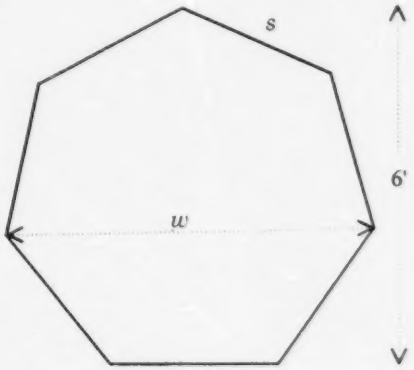
6. Sketch a rectangle in a semicircle as shown. What are the dimensions of the rectangle with the largest area possible if the radius of the circle is 5 cm? What is the largest possible area?



7. A bridge is being designed to span a small canyon as shown in the diagram. The arch of the bridge will be built as a 50° arc of a circle, and the roadway (a chord for the 50° arc) will be 22 metres long. Determine the length of the arc and the maximum height of the arch above the road.



8. A window is being designed as a regular heptagon (seven-sided polygon) where each sector of the window represents one day of the week. The window has a maximum height of 6 feet. Determine the side length, s, of each sector of the window, and also the maximum width, w, of the window. Write your answers rounded to the nearest 1/16 of an inch.



Exercise 3: Problem Solving Using Circle and Polygon Properties (continued)

9. The tiling of the floor of the Guyzoloni Palace is to be constructed of pentagonal tiles made of cast veredanium. You have to draw the master diagram from which the tiles will be cast. The tiles will then be used to construct the tessellating flooring.

   The chief designer has given you the following set of instructions for making the design:

   Step 1: Construct the segment AB, 10 cm long, with midpoint M.

   Step 2: On the same side of segment AB, construct two 45° angles with vertex M. Label the angles ∠ AMX and ∠ BMY.

   Step 3: Locate C along ray MY such that C lies on the circle having centre B and radius AB.

   Step 4: Locate E along ray MX such that E lies on the circle having centre A and radius AB.

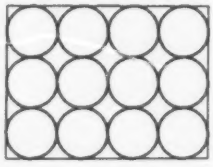
   Step 5: Locate D, the intersection of the circle having centre C and radius AB and the circle having centre E and radius AB.

   Step 6: Pentagon ABCDE is the required shape.

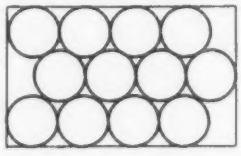
   Before you go any further, check that all the sides are the same length and measure all the angles of the pentagon. Notice that they are not all the same, so this is not a regular pentagon.

   Make 20 copies of your pentagon. Cut them out and use them to form a tessellation that will be the plan for the required flooring.

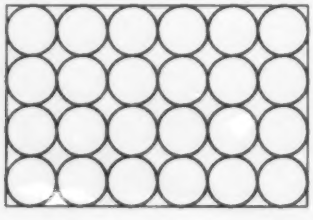
10. a) Canned goods, such as soup, are frequently packed 12 cans per carton, as shown. What is the total surface area of the carton if the cans are 10 cm high and have a diameter of 7 cm? The carton is just large enough to hold the cans.

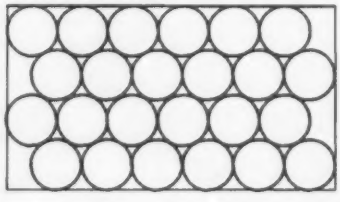


   b) A progressive carton manufacturer decides to explore new packing patterns, and needs your help to determine the surface area of the newly designed carton. What is the total surface area of the carton if the 12 cans were packaged as shown?



   c) Repeat questions (a) and (b) for cartons designed to hold 24 cans. The packing patterns are shown below. The type of packing on the left is **square packing**, and on the right is **hexagonal packing**.





   d) Which packing pattern is more efficient for 24 cans? What might be some considerations other than surface area of the carton when selecting a suitable design? Can you think of any packing patterns that are more efficient than the ones shown? If so, what are they?

Exercise 3: Problem Solving Using Circle and Polygon Properties (continued)

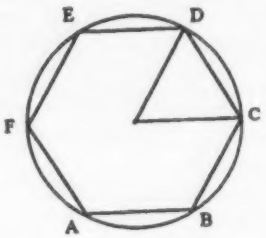
11. Archimedes: We have known since ancient times that the circumference of a circle has a measure a little more than three times its diameter. The exact relationship is called π (pi).

    Archimedes (BC 225) was the first to find a way to calculate π as precisely as we please. He began with a regular hexagon.

    a) If the radius is 1, what is the diameter? (*Answer:* 2)

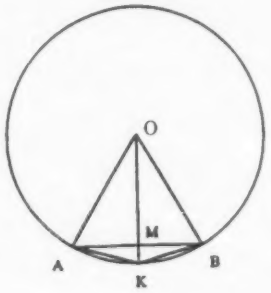
    b) What is the circumference of the hexagon? (*Answer:* 6)

    c) Using these two numbers (not the actual circumference of the circle, what is the first estimate of π?



    Archimedes then focused on one side.

    He drew a further radius through M, the midpoint of AB, to meet the circle at K.



    Note that OA = OB = OK = AB = 1 in this case.

    Later AB will be less.

    He then drew AK and KB.

    Use the Pythagorean theorem and a calculator to find OM and then MK. Then use MB and MK to find KB.

    AK and KB are two of the sides of a new polygon.

    How many sides does this new polygon have?

    Is this new polygon closer to the circle than the original hexagon?

    Using this new polygon, what is the estimate of π?

    Archimedes then shifted his attention to triangle OKB and did it all again.

    Since you know KB, what is NB?

    Find NL and LB.

    Estimate π again.

    How long could we keep doing this?

    Archimedes kept going until he had 96 sides (and he did not have a calculator). Go one step more than Archimedes (to 192 sides). Compare your estimate of π with the textbook value.

Unit H
Geometry
Solutions

Exercise 2: Circle and Polygon Investigations — Solutions

1. a) AC = BC, equal radii
   b) Δ ADC = Δ BDC, SSS or SAS
   c) Since $\angle 1$ and $\angle 2$ are corresponding angles of congruent triangles, $\angle 1 = \angle 2$. $\angle 1$ and $\angle 2$ are both congruent and supplementary and they are both 90°, therefore DC $\perp$ AB.
   d) Every point on the perpendicular bisector of a chord is equidistant from the endpoints of the chord.

2. a) 3
   b) 6
   c) 5
   d) 10

3. a) 6
   b) 4
   c) $2\sqrt{5}$

4. a) $4\sqrt{2}$
   b) 8
   c) $8\sqrt{2}$
   d) $4\sqrt{2}$

5. a) 130°
   b) 60°
   c) The inscribed angle equals one-half of central angle.

6. a) $\angle 3 = 22°$
   b) The inscribed angle equals one-half of the central angle.

7. a) $\angle$ BOD = $2x$
   b) $\angle$ COD = $2y$
   c) $x + y$
   d) $2x + 2y$

8. 40°

9. a) 54°, central angle equals twice inscribed angle
   b) 27°, inscribed angles subtended by the same arc are equal

10. a) Both are 46°.
    b) Both are 40°.

11. a) 59°, inscribed angle is half of central angle
    b) 59°

Exercise 2: Circle and Polygon Investigations — Solutions (continued)

12. a) 90°
    b) 13
    c) 6.5

13. a) 10
    b) 8

14. a) 13
    b) 12
    c) 30
    d) $\frac{169\pi}{4}$

15. Pythagoras: AB = $\sqrt{a^2+b^2}$ so radius = $\frac{\sqrt{a^2+b^2}}{2}$ and area = $\left(\frac{a^2+b^2}{4}\right)\pi$

16. 90°

17. a) 180°
    b) 90°, inscribed angle is one-half of central angle

18. ∠ C = 112°, ∠ B = 60°, ∠ D = 120°

19. ∠ ABC = 60°, ∠ A = 110°, ∠ D = 120°

20. ∠ ORQ = 60°, ∠ PQR = 88°, ∠ S = 92°, ∠ P = 85°

21. 41°

22. 45°

23. $5\sqrt{15}$

24. Since AD is a tangent and OC is a radius, ∠ OCD = 90°. It is given that ∠ ODC = 40°. Since the sum of the three angles in Δ OCD totals 180°, that leaves 50° or ∠ 1.

25. ∠ 2 = 100°

26. 105°

27. OP = 17, OR = 8, RP = 15

28. 8.7

29. 1440°

30. 2160°

31. 18 000°

32. $180(n-2)$

33. 8 sides

Exercise 2: Circle and Polygon Investigations — Solutions (continued)

34. 27 sides

35. $\frac{S}{180}+2$

36. a) 39°
    b) 102°
    c) 51°
    d) 102°
    e) 78°
    f) 51°
    g) 39°

Exercise 3: Problem Solving Using Circle and Polygon Properties — Solutions

1. b) The relation is $y = 0.5x^2 - 0.5x$ where x = number of points on the circle and y = number of chords.

2.

x	1	2	3	4	5	6
y	1	2	4	8	16	31

The relation appears to be $y = 2^x$ where y = number of regions and x = the number of points on the circle. The relation breaks down when $x = 6$ points.

3. (Answers may vary depending on the size of the diagram.) Construct two chords for the arc shown. Draw the perpendicular bisectors of each chord. The point of intersection of the two chords is the centre of the circle. Measure one radius of the circle, and multiply by 2 to get the diameter.
4. The three chords will be congruent (that is, the three chords will intersect at a common point).
5. The sums of the opposite side lengths are equal, i.e., EF + GH = FG + HE.
6. Length = 7.1 cm, width = 3.5 cm, area = 25 cm^2
8. Side length = 2 ft. $8\frac{7}{8}$ in; maximum width = 6ft $1\frac{7}{8}$ in
9. a) Surface area = 2156 cm

   b) Surface area = 2217.3 cm

   c) Rectangular packing surface area = 3572 cm, hexagonal surface area = 3705.7 cm

   d) Hexagonal packing is more efficient when considering the amount of material used to construct a carton. The saving is about 1.2%.